The Poincaré Institute:
Nurturing Expertise in Mathematics Teaching and Learning

Tufts University, TERC, and Nine School Districts in Maine (Portland),
New Hampshire (Dover, Timberlane and Sanborn) and
Massachusetts (Fitchburg, Leominster, Medford, Medway, and Somerville)

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Summary

The Poincaré Institute aims to promote student learning of Middle School mathematics by developing teachers' expertise regarding (1) key mathematical objects related to algebra and the mathematics of functions; (2) representational systems for these objects (tables, number lines, graphs, written notation, and natural language); (3) modeling situations involving quantities and (4) classroom research about students' mathematical representations. Teachers (grades 5-9) from nine New England districts will engage in online courses as well as discussion forums in schools. Teacher and student progress will be evaluated on the basis of classroom discussions, interviews, and written assessments. Dependent measures of student success cover their mastery of mathematical objects and their properties (functions, equations, equivalence), algebra in modeling, and using and interpreting mathematical representations.

I. Questions for Dialogue

- How should we evaluate student success in learning mathematics?
- Are written assessments enough? What else could capture deep mathematical understanding? Interviews? Classroom observations? How?
- What about what happens beyond the school year? Or outside of schools? What kind of funding is needed to evaluate student success beyond a funded intervention's place and timeline?

II. Conceptual Framework: STEM education and our MSP Project

Middle school is a critical period during which students are expanding their knowledge about numbers and number systems, using algebraic notation and graphs to represent intra- and extra-mathematical relations, moving beyond operations on *particular numbers and measures* to operations on *sets of numbers and measures*, and operating on expressions containing *variables*. These transitions are vital to student success, yet this is also a point at which many students lose interest in mathematics. The problem is acute not only in urban centers, where algebra has often served as an "engine of inequity" that widens the achievement gap between socio-economic, racial, and ethnic groups (Kaput, 1998), but is also a concern in rural areas and in areas with a high concentration of immigrants. Partners in this proposal—mathematicians, physicists, education researchers, school teachers, and administrators—share a commitment to improving the teaching and learning of middle school mathematics and to promoting smooth transitions across the mathematics of elementary, middle, and high school.

We propose not to re-teach the mathematics the teachers already know, nor to present pre-packaged lessons that can be directly implemented in their classrooms, but rather to offer a fresh, inquiry-based, multidisciplinary course of investigation and framework for re-envisioning the mathematics they already teach. We chose algebra and the mathematics of functions as the core organizing subjects in order to unite topics usually taught in isolation, such as arithmetical operations, fractions, ratio, and proportion. For scientists, functions are perhaps *the* key mathematical tool for modeling properties and processes of the physical world. We will draw upon a large body of research in mathematics education—for instance, work on conceptual fields in mathematics education (Vergnaud, 1996), algebra and the mathematics of functions (Chazan, 1996; Dubinsky & Harel, 1992; Harel & Confrey, 1994; Schwartz, 2008; Schwartz & Yerushalmy, 1992, 1995; Yerushalmy, 2005), and work on algebra in elementary and middle school (e.g. Kaput, Carraher, & Blanton, 2007; Schliemann, Carraher, & Brizuela, 2007, in press).

The partnership has several innovative characteristics, four of which are mentioned here:

Interdisciplinary cross-fertilization. The activities that are presented to teachers on each week of the course have been planned by mathematicians, scientists, and mathematics education researchers.

Second takes. The online courses are highly interactive, centering on the teachers' responses to two weekly challenges. The first challenge in each week is designed to be open-ended and promote exploration, speculation and discussion. Teachers answer the first challenge before having access to all readings, notes, videotapes, and other materials. After considering extra readings, notes, videotapes of classroom episodes and interviews with students, and the views of specialists and of other teachers in the course, they answer the second challenge, which may include or elaborate on some of the questions from the first challenge. This allows course instructors to look at the evolution of teachers' work and gives teachers the opportunity to note how their thinking evolves.

Discussion of student work. The Institute draws on a 12-year NSF-funded program of classroom research about how students in grades 3-8 develop algebraic reasoning. Teachers solve and analyze problems and speculate about how middle school students would approach these problems. They then watch videos of classroom episodes and interviews with students and examine how students actually solve similar problems.

Continuing discussion groups in their schools. Poincaré teachers and colleagues hold regular discussions about their own mathematics classrooms. They work together to plan, implement, videotape and discuss their own lessons and their students' learning.

Three cohorts of 60 teachers each will participate in the sequence of three Poincaré online courses offered over three semesters. Partner districts represent a variety of sizes and locations and have participated in discussions about the design and implementation of the project.

The project rests on the premise that to improve students' learning one needs to broaden and deepen teachers' understanding *of mathematical content, of how children think and learn, and of mathematics knowledge for teaching*. We describe the growth in mathematics education expertise that we are aiming to promote among Poincaré teachers and the impact we hypothesize it will have on their middle school students' mathematics learning.

To wager on the short-term impact of an MSP program for teachers on their middle school students is a decidedly risky venture. It is predicated on the assumptions that (a) the program will have a significant impact on the teachers and (b) that this impact on teachers will lead quickly to positive, measurable improvement among the students. This might be easily achieved in the case of highly circumscribed and specific skills for example, computational proficiency with fractions. We could provide teachers with intense practice in such matters and teach them how to use similar exercises with their students. It would then be a straightforward matter to evaluate the students for proficiency in computing with fractions.

Even though we will analyze student success in terms of proficiency in specific skills, at the Poincaré Institute we work towards promoting and evaluating students' deep understanding of mathematical content and use of algebra and functions as modeling tools to solve problems in multiple contexts. Analysis of computation proficiency as well as a certain level of mathematical understanding related to algebra and functions may be relatively straightforward in the case of written assessments. Data from written assessments, however, will need to be complemented with deeper probing of students' understanding. In this project we will be specially interested in demonstrations of students' mathematical understanding as they participate in lessons and in classroom discussions. Three overarching aspects of their participation in lessons will guide our analysis. They are:

(a) Use of algebra as a modeling tool in extra-mathematical contexts,
(b) Use of multiple representations for functions, and
(c) Willingness to explore problems in depth, considering all potentially relevant aspects before proposing solution methods and answers.

We consider that understanding of algebra and functions as demonstrated in written assessments and the in the above three aspects are indicators of student success in learning mathematics in middle school.

In our analysis of the impact of teachers' participation in the Poincaré Institute *on students' learning and success*, we hypothesize that:

1. Students in classrooms of teachers who have taken the three online courses will perform significantly better than students of teachers who had not yet taken the courses in each of the proposed written assessments (MCAS or NECAP and the Poincaré Institute's Assessment).
2. Videotaped classroom lessons taught by the same teacher, after they take the three online courses, will show that their students engage in deeper explorations of mathematical relations, use multiple representation systems, more frequently use algebra as a tool for modeling science and everyday problems and situations than was the case in videotaped lessons taught before teachers took the courses.
3. District student results in each of the proposed measures (MCAS or NECAP and the Poincaré Institute's Assessment) will be significantly better at the end of the project, in comparison to those at the beginning of the project.

III. Explanatory Framework: Research design, data collection, and analysis

Teachers in the first cohort will take the three courses from January 2011 to May 2012. Those in the second cohort, from September 2012 to December 2013. And teachers in the third cohort, from January 2014 to May 2015. All teachers are encouraged to participate in discussion forum meetings from January 2011 and thereafter. Data on their students will be collected as follows:

- Students' state-mandated assessment results (MCAS or NECAP),
- Students' results in assessments designed by the project team,
- Students' activities, discussions, responses to researchers' questions, and written work during a sample of videotaped lessons.

Assessment data and videotaped lessons will be collected at the start and end of the five-year project and, for teachers in each cohort, at the start and end of each three-course sequence. This will allow analysis of overall progress and of progress related to participation in online courses, as required for testing our hypotheses.

The impact of the Poincaré Institute's activities will be analyzed in terms of teachers' implementation of effective teaching activities and of teachers' and students' evolving understanding of mathematical content and representations regarding:

(a) Numbers, Fractions, Relations, Linear and Non-Linear Functions, and Algebraic Equations;
(b) Multiple representation systems: Natural Language, Tables, the Real Number Line, Parallel number line representations, Graphs in the Cartesian Space, Algebra Notation.

Comparisons between pre-and post-written assessment measures will allow for evaluation of the impact of teacher development on student success as measured by written assessments.

Detailed qualitative analysis of students' questions, answers, argumentation, justifications, solutions, and written work, as they participate in videotaped lessons before and after their teachers are taking courses, will allow for insights into how teachers' ways of teaching and increasing understanding of mathematics and mathematics teaching may impact student success in terms of their learning and deeper understanding of mathematics for representing and solving more complex problems and problems related to everyday situations and to science content.

During the presentation at the January 23, 2011 PI meeting, we will present a subset of the items we have selected to evaluate student progress. We will describe the rationale for their inclusion in the research and their relation to the conceptual framework and content of the Poincaré Institute courses.

Key prospective insights

We need to pay special attention to the evidentiary basis of the research by raising the following questions:

- What data will serve as evidence of **teacher progress** (or lack thereof)? Will we be able to attribute any observed changes in teacher performance to their participation in the Poincaré Institute courses and other activities?
- What data will serve as evidence of **student progress** (or lack thereof)? Will we be able to attribute any observed changes in student performance to the fact that their teachers took part in the Poincaré Institute courses?

These questions arise from concerns about internal reliability and validity. They are addressed to the extent that the project's design will allow us to decide whether any changes observed are not due to familiarity with the assessment items, learning and development not related to the Poincaré intervention, or other extraneous factors.

We need to consider matters of experimental design and issues of construct validity such as:

- How can we be sure that our courses and "extracurricular" activities will deal with the kinds of expertise we are interested in promoting?
- How can we be sure that our student measures will capture the sort of gains we expect to see among students?

The combination of quantitative and qualitative analyses envisioned by the Poincaré Institute interdisciplinary team will hopefully provide data that will interrelate teachers' experiences in the courses and discussions to changes in their mathematical understanding, their ways of teaching, and their students' success in mathematics.

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