Title: AMP: Professional development for improving middle school teacher and student problem-solving skills.

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Abstract: This article provides grounded insights about professional development experiences that promote teachers' and students' growth of productive problem-solving behaviors. Artifacts and use of teacher-generated products provide evidence of teacher growth (as described by Lischka, Sanchez, Kastberg & Tyminski, 2016) in orchestrating appropriate learning experiences for students and teachers' ability to identify specific aspects of student problem-solving growth. Teacher growth in problem solving is recognized ultimately to be cyclical rather than linear and is profoundly affected by teachers' workshop experiences and subsequent reflections. Facilitator and teacher resources are included.

The mathematical community currently has access to a well-crafted set of content standards and mathematical practices from Common Core State Standards of Mathematics (CCSSM) published in 2010 and effective teaching practices from the National Council of Teachers of Mathematics (NCTM, 2014) publication *Principles to Action*, that collectively are designed to raise the United States' quality of K-12 mathematics with regard to international competiveness but also to improve the college and career readiness of today's students. These documents however "do not tell teachers … how to begin making essential changes to implement the standards" (NCTM, 2014). Two additional factors make the present report compelling: 1) the majority of states use CCSSM or adaptations with significant attention placed on problem solving (Sztajn, Marrongelle, Smith, & Melton, 2012); and 2) a predicted shortage of STEM teachers through 2025 due to a decline in teacher preparation enrollments, higher student-teacher ratios, and higher teacher attrition (Sutcher, Darling-Hammond, & Carver-Thomas, 2016).

The threefold purpose of this paper is to provide resources to facilitators and coaches for improving teachers' skills in nurturing and diagnosing students' problem-solving behaviors, to share teacher-generated classroom innovations that correspond to their experiences in a two-year professional development project, and to increase awareness of issues related to improving teachers' and students' problem-solving skills in a format that is accessible to all audiences. This article is based on a synthesis of findings from a series of design experiments (as described by Schoenfeld (2014) and Cobb et al. (2003)) conducted over the last ten years for the purpose of having research inform teacher training as well as students' mathematical performance with a focus on problem-solving. Data has been triangulated from filmed teacher-interviews, classroom observations, teacher reflections, third-party evaluator reports and samples of teacher and student work

What We Know About Professional Development

In their recommendations for mathematics teachers' professional development (PD) , Sztajn et al. (2012) claim the Mathematical Practices included in the CCSSM (such as sense-making, constructing viable arguments, critiquing the reasoning of others, attending to precision, perseverance and communicating ideas) define what desired teaching and learning should look like. Mullins et al. (2012-2013) identify four reasons the Mathematical Practices have received less attention than Content standards. These include (a) teachers and administrators do not understand what some of the mathematical practices are trying to describe, (b) many teachers were taught in traditional lecture style and have never experienced learning in an environment focused on developing the mathematical practices (Mayer et al. (2011), (c) teachers struggle to envision what classrooms would look like where students learn content through engaging in the Standards for Mathematical Practice, and (d) many administrators and teachers focus on the Mathematical Content as the way to raise test scores and see the Standards for Mathematical Practice as less essential (p. 31).

Darling-Hammond and Richardson (2009) claim that effective professional learning in general should (a) be content based and include pedagogical issues of teaching and learning of that content, (b) use active learning, (c) include reflection with colleagues, (d) be consistent with school, district, and state reforms, and (e) be intensive and sustained over time.

Recent studies have established that through effective professional development teachers can change their beliefs about teaching and learning, improve their own mathematics and ultimately change their classroom practices (Hsu, Kysh, Resek, & Ramage, 2012; Lebak &

Shule, 2014). However, changes in teaching practice takes time and meaningful change can take years (Jaberg, Lubinski & Yazujian, 2002).

Clarke and Hollingsworth (2002) (See Figure 1) report that teacher change is personal and follows a non-linear cycle through various domains that may include workshop experiences, classroom implementation, assessment, interaction with colleagues and/or workshop facilitators and reflections made during and amongst those various experiences. Gresalfi and Cobb (2011) claim that a teacher's active participation in PD will profoundly shape the norms, values and practices in her/his professional activity. Sztajn et al. (2012) recommend that effective PD requires that teachers' engage in constructing viable arguments and other practices that should occur in their own classrooms.

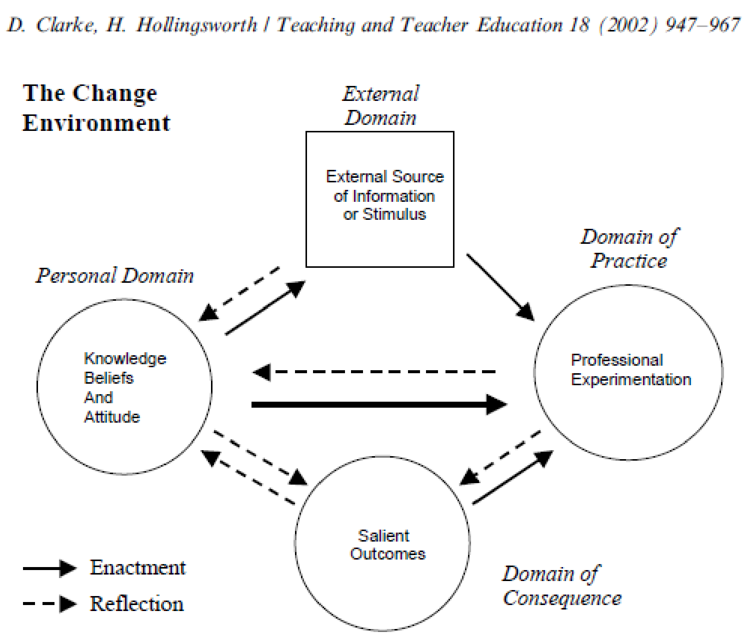


Figure 1: Clarke & Hollingsworth (2002) Model of Teacher Change.

The primary goal of Arizona Mathematics Partnership (AMP), the six-year, National Science Foundation funded project was to increase student achievement in middle school mathematics by first improving teachers' content knowledge, problem solving ability and their classroom practices. Workshop hours focused on improving teachers' mathematical content knowledge, awareness and facility with issues of students' understanding, classroom discourse and problem solving. The Community of Collaborative Learners (CCOL) hours focused on local campus needs and the helping teachers deal with their own students' learning needs. AMP devoted approximately 25% of workshop contact hours to problem solving and the related issues of teaching and learning problem solving, the remainder of time was spent on developing the content and pedagogical knowledge of teachers. Over a two-year period each participant received a total of 162 hours of PD: (a) 35 hours per summer institute, (b) 28 hours of Saturday workshop hours per year, and (c) 18 hours of CCOL hours per year.

Teachers were instructed on their first day that they would wear two hats, one as a student actively engaged in problem solving and related activities as described by Woodward et al. (2012), including (a) preparing routine and non-routine problems and use them in whole-class instruction, (b) assisting students in monitoring and reflecting on the problem-solving process by providing prompts and modeling how to monitor progress, (c) teaching students how to use visual representations by using think-alouds and proper notation, (d) exposing students to multiple problem-solving strategies and provide opportunities for students to compare multiple strategies in worked examples, and (e) helping students recognize and articulate mathematical concepts and notation and relate them to problem-solving activity; and the other as a reflective practitioner thinking deeply about the experiences that create and nurture a mathematics classroom culture (Schoenfeld, 1992; 2014) in which students have the experience of doing mathematics with attention to (a) development of a mathematical point of view – using mathematics to symbolize, abstract, model, prove or disprove conjectures, perceiving connections across problems and results, and creating knowledge that is new to oneself or the community, (b) emphasis on process as well as results where explanations of how ideas are generated are highly valued even when they do not produce solutions, (c) authority - the teacher leads the class towards assuming responsibility for standards of completeness, coherence, and the conviction of mathematical arguments and where the mathematics itself is the ultimate authority (d) communication - the classroom setting encourages written and oral communication where ideas, not the person, are critiqued, and (e) reflective mathematical practice – by asking "Is your argument convincing? How could you arrive at the same answer using a different solution pathway? Can this result be generalized?"

Enacted Professional Development Design of AMP Problem-Solving Workshops

The content of the AMP problem-solving (PS) workshop experiences were based on Schoenfeld's (1993; 2013) theoretical framework for describing problem-solving behaviors that includes the problem solver's resource knowledge, planning, appropriate strategy selection, (metacognitive) monitoring of progress, verification and belief system. Additionally, the teacher's ability to diagnose a student's productive problem-solving behavior was guided by Vicich's (2014) Diagnostic Instrument (see Appendix 1). Task selection was guided by several criteria including that problems (a) were open-ended and served as platforms for exploring mathematical relationships, (b) served as opportunities for the facilitator to illustrate/model the mathematical practices and effective teaching practices, and (c) focused on each category of productive PS described in the Diagnostic Instrument. To create teachers' empathy for students' difficulties with problem solving the degree of difficulty was chosen to reside at the limit of middle school teachers' curricular boundaries which was taken to include ninth and tenth grade mathematics and the vertical and horizontal connections that exist from grades 4 through 10 as described in the CCSSM. Access to problem-solving materials for both facilitator and teacher resources are available at http://amp.azmath.org .

Initial Theory and Actions for Increasing Teachers' Awareness of Problem-Solving Behaviors

Simply stated, the facilitator's initial theory for helping teachers increase their problem-solving skills and abilities to help students become better problem solvers was to simultaneously (a) introduce epistemological components of frameworks and research that support productive problem-solving behaviors with teachers playing the role of researcher and subject, and (b) create episodes of good teaching within workshops that model implementation of the CCSSM Mathematical Practices and the NCTM Teaching Practices.

The AMP facilitator team was aware that a previous Mathematics Science Partnership study conducted in Arizona (C-Cubed, 2008) showed that middle school mathematics teachers had (a) limited understanding of productive problem-solving behaviors (most often a variation of Polya's (1981) four-step process: Understand the problem; Devise a plan; Carry out the plan; Look back.), (b) limited awareness of unproductive beliefs, and (c) limited knowledge of problem solving strategies and heuristics.

As a means of enabling teachers to identify productive problem-solving behaviors, teachers were asked to act as a researcher observing/listening to another's problem-solving behaviors by using the Diagnostic Instrument (Appendix 1). Teachers were paired as problem solver and observer. Solvers were asked to think aloud while observers commented on the various categories of behavior such as initial engagement, planning, conjecturing, strategy selection, monitoring of progress, verification, resource knowledge, and beliefs and attitudes such as willingness to explore and perseverance. Teachers then presented their solutions to the larger audience by white board or document camera. Answers were to be written in complete sentences using proper grammar and appropriate units. Audience members critiqued the mathematics and the observer shared her/his observations as they unfolded. While solving several problems the roles were reversed. In this activity, teachers developed 1) awareness of their own problem-solving behaviors; 2) awareness of how to diagnose students' difficulties; and 3) empathy for the students' experience as problem solver. Teacher reflections after this activity included the benefit of seeing alternative solution pathways and the power of accurately identifying givens and goals.

Teachers solved problems that used various strategies from a comprehensive list adapted from Schoenfeld (1998, 1980). These strategies included (a) draw and label a diagram if possible, (b) examine special cases, (c) select specific values to get a "feel" for the problem, (d) exploit both *result* and *method* of similar problems, (e) act out the problem or use manipulatives, and (f) draw an auxiliary line and decompose complicated figures into component parts.

The facilitator of the problem-solving workshop sessions created a safe environment in which the mathematics itself was critiqued, rather than the person presenting her/his ideas. Teacher participants took on the role of student within a student-centered classroom culture. The workshop facilitators modeled teacher behaviors that were designed to promote, support and establish expectations for student behaviors/actions (adapted from AMP Sustainability Factors available at http://amp.azmath.org/Resource/pdondemand). These expectations included that (a) students will be engaged in mathematical discourse with the teacher and with fellow students, (b) students will be explaining their mathematical thinking, (c) students will be justifying their reasoning mathematically, (d) students will critique the thinking of others, (e) mathematical reasoning will be the ultimate authority to determine the validity of a claim and students will need to convince their own classmates through sound and logical reasoning, (f) students will be engaged in problem-solving, (g) students will be communicating (verbally and in writing) mathematical thinking coherently and precisely (language), (h) students will be sharing alternative solution pathways, and (i) students will embrace mistakes and remain persistent when problem solving.

Two Workshop Episodes of Teachers' PD Experiences

On the first Monday of the summer workshop teachers were assigned **The Ship Problem** (Downs, 1993) and they had all week to complete the problem. Most teachers drew pictures, created charts and looked for patterns. With N = 82 total teachers, 45 received 'C' for correct; 10 received 'CM' correct with minor errors and notes for improvement; and 27 received 'R' for Resubmit. The purpose of the problem was to have teachers make sense of a real-world situation, create a mathematical model, and persevere while engaging in productive struggle over an extended period of time.

This problem created initial frustration among the majority of teachers (as there is no clear algorithm available) and some teachers fell back on to manipulating concrete objects to make sense of the givens (schedule of ships leaving harbor) and goal (determining the number of open sea crossings).

The Ship Problem

(Circa 1875): Twelve ships will leave San Francisco, one per month, to travel around the horn of South America bound for New York. At the same time, twelve different ships will leave New York, one per month, along the same route bound for San Francisco. Excluding meetings in the harbors, how many times will ships headed in opposite directions pass each other on the open seas? Each ship will take six months to reach its destination. Make a visual representation of the given information.

For one teacher, Julia (pseudonym), the Ship Problem created a new awareness of the kind of problem that could challenge students to think about mathematics outside of her classroom.

[Julia's background: BS Elem. Ed. and a MS in Elem. Ed. Curriculum and Instruction. 11 years total teaching experience, 5 years teaching sixth grade, and 6 years teaching third and fifth grade.]

Author Question: You mentioned that the ship problem shifted your beliefs and practices, how would I see those changes today compared to "before" AMP?

Julia's Response: "Prior to AMP, I had been wanting to send home better, more meaningful homework that enabled my students to apply the concepts from class, but I wasn't sure how to align my beliefs with best practices. I was sending home procedural work from the required textbook, but something was missing. The ship problem gave me the "how;" I realized how I could create more meaningful tasks for my students by providing better questions that included a productive struggle and focused on multiple math practices. Problems with multiple entry points, such as the ship problem, differentiate, provide opportunities for purposeful discourse, and allow students the time to engage in a meaningful task."

Figure 2: Teacher Interview (Ship Problem).

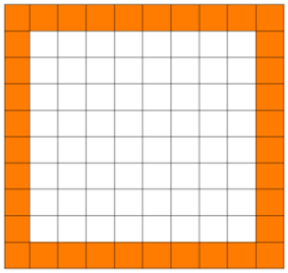
The solution to the Ship Problem can be efficiently verified by finding the complement (that is, identify the number of non-open sea crossings) to the goal. In the subsequent Saturday workshop, sharing the solution pathway of finding the complement became a meaningful learning experience for teachers as many teachers had spent several hours using their initial solution pathway, searching for patterns or resorting to manipulating concrete objects.

Also during the summer week the **Border Problem** was used to model effective teaching practices that promote respectful classroom discourse. The facilitator asked for ways of thinking about the answer, for example, "I counted four sides of ten and then subtracted the four corners." Or "I added the top and bottom rows of ten and then added the left eight and right eight." The facilitator then wrote the name of the person next to the method, such as Cindy's Method. Several methods for arriving at the answer were demonstrated. As the problem moved toward the N by N generalization several teachers had shifted from their original approach to another that they deemed to be more efficient. The teachers then watched the video (available at https://www.youtube.com/watch?v=PKkCqtRVm90 Retrieved 5/25/17) of a teacher with her eighth-grade students repeating the same leaning experience.

The Border Problem

(https://themathletes.wordpress.com/2013/10/07/the-borderproblem/ Retrieved 4/6/15)

Without counting one by one, determine how many shaded squares are in this 10×10 grid.



What about a 6 in by 6 in grid? What about a 15 in by 15 in grid?

What about a 253 in by 253 in grid? What about an *n* inch by *n* inch grid?

During interviews, one sixth grade teacher, Susan (pseudonym), had expressed a shift in her belief system about teaching and learning mathematics as she reflected on the Border Problem.

[Susan: BA Elem. Ed.; MS Elem. Ed.; National Board for Professional Teaching Standards Certification; 10 years teaching third, fourth and fifth grades , 2 years teaching sixth grade; and 2 years teaching seventh grade.]

"The Border Problem hit home with me during the first summer week. I never really considered that solving problems in more than one way is allowed and useful. I like that problem because it is simplified, it is not a hard problem, and everyone can access it. My Ah-Ha moment was that students should solve problems in the way that makes the most sense to them and also be able to critique/understand other methods. I use this activity at the beginning of the year now to demonstrate this to students. I made a task out of it, I have a sheet they can record on (and) I tell them 'however this made sense to you we are going to share so everyone else can make sense of it'. They LOVE the freedom of not being confined to one method and enjoy coming up with the creative alternatives."

Figure 3: Teacher Interview (Border Problem).

Teacher-Generated Classroom Innovations

Two outcomes of the summer workshop experiences were the teacher-generated products called the Camera-Ready Expectations (and posters) and the Problem-Solving Template (PST).

Teachers were regularly called on by the facilitator to share their work at the document camera (as the workshop campus had limited board space). The following expectations for the work, called “Camera-Ready” by the facilitator were used as talking points for peer feedback.

The facilitator always began critiques with “What did you like about her/his presentation?” to give positive feedback to all presenters whether they answered the problem correctly or not. Samples of peer comments included “I liked that she showed every step in her thinking’, or “I liked his use of precise units and labels.” If improvements could be made the facilitator asked audience members,” How can he/she put the plus on his/her A?” Vicich and Clark (2016) noted that the phrase ‘**Camera-Ready’** became a “taken-as-shared mathematical practice established by the classroom community” as described by Cobb and Yackel (1998). AMP teachers use the phrase ‘Camera-Ready’ in their normal classroom discourse and some have posters they have created displayed in their classroom (See Appendix 2 for Julia’s Poster). “The poster is a combination of my AMP experiences and the 7c's of creativity used at [my school] by teachers and students. The 7cs are: cogitate, collaborate, calculate, craft, construct, communicate, and connect. ” Note: The word ‘craft’ for Julia meant the art of communicating.

Oral presentations made with the aid of a doc camera offered several opportunities for shared practices. These included use of precise language and vocabulary, use of proper notation, a verification component in a solution, reinforcement of common procedural skills and algorithms but also opportunities to present alternative solution pathways that may have been clever or elegant. Often times teachers remarked when seeing an alternative solution pathway, "I didn't think of solving the problem that way myself."

At the end of summer week one, two teachers shared their first versions of a **Problem-Solving Template** (PST) designed to provide structure with all other participants. The categories of problem-solving behaviors of *identify given and goals, make plans and conjectures, justification, final answer as a complete sentence and verification* were organized into boxes on a two-sided handout. The facilitator intervened and suggested including a grading rubric for giving students feedback. The rubric would communicate the value of developing productive behaviors and habits of mind without necessarily getting the final answer correct. A second generation of PSTs that included grading rubrics emerged and was shared at subsequent workshops. A third generation seen in Appendix 3 included strategies, and guiding questions to help students make progress.

In a recent survey of AMP participants (n = 81), 95% of teachers reported using some form of the PST during the school year. 69% of those using a PST preferred to use it more frequently at the beginning of year to establish quality of work expectations and "to get them thinking like a problem solver." Teachers used the PST throughout the year to "help students maintain productive habits as well as promote organizational skills and consistency in their work. Some teachers had blank templates available throughout the year and other teachers prefer that students "eventually create camera-ready work without it."

Although the Problem Solving Templates and use of Camera-Ready expectations for student work emerged during first two years of the project an additional attempt to increase and assess teachers' acumen with analyzing students' problem-solving behaviors, the facilitator created an activity called the Problem-Solving Portfolio that was initiated with the third and fourth cohorts. See Appendix 5 for Portfolio instructions.

Portfolio Activity

Teachers were required to type a one-page (max) Student Growth Summary & Analysis for three students (low, average and high performing) solving three different problems selected from regular class work during the year, addressing the categories of PS behavior listed in the Diagnostic Instrument. See Appendix 4 for a sample of student work and Susan's analysis of the students' growth during the year. Susan found the portfolio activity to be beneficial:

> The biggest take-away for me through this portfolio process was the forced reflection on my students' progress and the realization of how far they truly have come since the beginning of the year … the insight I have gained into each and every one of my student's problem-solving behaviors and overall mathematical ability is tremendous. I have realized that I know these students better than any of the other students I have taught math to in the past. This has allowed me to be a better teacher because I am more aware of their strengths, weaknesses and most importantly, their individual ways of approaching mathematical situations. I have never before had so much information about how my students think when it comes to math and, therefore, have never before been able to guide them as effectively as I have this year.

Collectively the quality of teachers' portfolio analyses of student work varied from minimal effort to quite detailed and insightful analysis. Some teachers' use of language revealed that teachers had comfort and familiarity with the categories of problem-solving behavior and the impact meaningful problem solving has on students' learning. For example, one teacher wrote:

> My students are learning that helping each other does not mean giving each other the answer. This problem (referencing a problem in her portfolio) was a test in perseverance, it was difficult to make a conjecture, but other students realized and shared the givens … the conversation was very student led!

Evidence of Teacher Growth

Lischka et al. (2016) describe criteria to provide evidence of prospective teacher growth (based on the three theoretical perspectives of socio-political, cognitive and situative). The following three criteria have been adapted to the present study for the purpose of reporting practices that support teacher growth:

Teachers use an activity from PD. Several AMP teachers used problems taken directly from workshop materials. They have experienced the materials and publicly shared their mathematical thinking in a student-centered classroom and report a sense of confidence in using the learning experience with their own students.

Residue or continued use of an activity. The Problem-Solving Template was used by many AMP teachers throughout the year to provide structure and practice in developing habits of mind and productive problem-solving behaviors.

Internalization of PD materials. The use of Camera-Ready expectations for students presenting their work at a doc-camera or board provided evidence that teachers had incorporated higher expectation of precision, precise vocabulary and the goal for students creating convincing, coherent solutions. In her classroom, Susan told students, "Your work is now for everybody not just yourself."

Refined Theory of PD Design and Interventions

The initial theory for AMP workshop design and interventions with a focus on problem solving was hypothesized to be linear rather than a *continual* cycle of experience and reflection. Providing teachers with a ‘research like’ experience to deepen their understanding of productive problem-solving behaviors, although ambitious, did not appear to be a complete learning experience, internalized by the teacher until the teacher had enacted a problem-solving learning experience with her/his own students. Additionally, the crowded schedule of teaching, grading and planning left little time for deeper reflection as described in Susan’s portfolio report. The PST and Camera Ready expectations served as vehicles for teachers to address the Mathematical Practices and Teaching Practices with a deeper understanding of the supporting epistemology (meaning the nature and methods of knowing) for improving one’s problem solving abilities. The Portfolio Activity provided teachers with the opportunity to develop a long-term history and analysis of their students’ problem-solving behaviors over four to five months. Teachers’ reflections contained in the Portfolio Activity approached the depth of awareness and ability to diagnose students’ problem-solving behaviors that were initially hypothesized to occur as a result of the workshop experiences alone.

A Call for Administrative Support

Excerpt from an AMP teacher (participant) email to facilitators:

> After a year of participation in AMP, the other participants [at my school] and myself have had a variety of administrative conflicts about our new methodology based on what

we're learning in AMP. For example, it is not uncommon to spend a sixty minute class period discussing and exploring in great depth a single problem from the day's objective. Student-centered discussions with outstanding math talk are common and documented by our CCOL mentor, with his compliments and praise. Feedback that we are receiving from our site administration is that we are wasting time with the students presenting their 'camera ready' work under the document camera and detailing their step-by-step thinking process. It is the opinion of our site administration that we should spend significantly less time per problem and increase the number of problems per class period. We have been told that having students go into such depth disengages the students and lowers the effectiveness and quality of the instruction because the students see fewer problems … Is there anything that can be done from AMP on our behalf to strengthen the administrative understanding of what we are attempting to do, such as professional development for administrators?"

Although the AMP Project hosted a yearly administrative workshop (that not only gave an overview of project goals but also required administrators to actively engage in solving a sampling of problems covered in teacher workshops) and provided an open invitation to attend all teacher-workshops, not every administrative leader was 'required' to change their underlying belief system to one that supported the type of mathematics instruction as described in this report. Teachers, administrative leaders and those responsible for designing and facilitating professional development must make a greater effort to identify and attend to each other's ultimate goals. In the cases of Julia and Susan, both teachers had very strong administrative support for their professional growth that recognized and valued the students' improved problem-solving behaviors.

Final Reflection and Take-Aways

A summary of the major recommendations that are grounded in the data of the AMP project includes that (a) workshop experiences (alone) using the Diagnostic Instrument for increasing teachers' understanding of productive problem-solving behaviors were insufficient for developing deeper teacher understanding and diagnostic skills, (b) teachers should analyze students' behaviors over time (say 4 – 5 months) using multiple student ability levels and multiple problem contexts within a framework such as the Portfolio Activity, (c) adequate opportunities must be provided during PD for deep teacher-reflections about students' problem-solving behaviors, problem selection and teachers' own classroom practices, (d) use of teacher-generated products such as Camera-Ready expectations for student presentations and variations of the Problem-Solving Template fostered teacher ownership of instructional experiences that were modeled in workshops, and (e) administrative support for the tenets of the PD Project was necessary for positive teacher growth to occur over time.

Finally, teachers' growth was generated by an interactive cycle of learning in workshops, classroom implementation of instruction modeled in the workshops and the reflections that occurred during and between those experiences. In this project teacher growth occurred over two years, using a variety of PD experiences and empowered teachers to change their previous classroom practices as evidenced in Susan's reflection:

> I have become a more effective teacher of mathematics in every way possible. My classroom is student-centered, rigorous, and focused on discourse as well as problem solving. My students are fully engaged and are learning from each other in ways they

> never imagined. This has led to much deeper conversations about mathematical practices and content than what I experienced previous to AMP.

Future projects must recognize that each teacher comes to professional development with unique learning needs and an underlying belief system (as revealed in Julia's and Susan's reaction to the Ship and Border Problems). Teachers can grow in positive ways over a sustained period of time with regard to their own problem solving skills and also in providing student-centered classroom instruction (as demonstrated by the Camera-Ready poster). Resources provided to teachers in workshops, such as the Problem-Solving Template, must be useful to teachers in their own classroom instruction. Facilitators must acknowledge the cyclic nature of teacher growth when designing professional development experiences for teachers. Workshops alone are not sufficient for increasing teachers' awareness of students' problem-solving behaviors but should be augmented with a long-term, guided investigation of students' work (such as the Portfolio Activity). In the aggregate AMP teachers showed growth in content knowledge and student scores on the state assessment (called AzMERIT) improved in the schools with highest teacher participation in the AMP program.

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APPENDIX 1 Diagnostic Instrument for Problem-Solving Behavior

(Vicich, 2014)(Adapted from Geiger & Galbraith, 1998)

Engagement

Problem is read	Key words underlined	Givens and goals established	Givens and goals represented symbolically

*Executive Behaviors

*****Planning: Did you make a *plan* or "jump into" this problem? Did you make any *conjectures regarding the answer or possible solution path*?

*****Monitoring/Control

Recognition that a solution pathway will lead to a dead end	Changing from one solution pathway to a different solution pathway

*****Heuristic Strategies

Appropriate strategy initially selected	Data organized	Multiple Strategies used to make progress or clarify	No heuristic used

Verification

Checked if answer was reasonable	checked correctness of answer	Checked for errors in solution	No verification used

Mathematical Practices and Habits: Solution (is)

Based on reason/logic	Thorough/Complete	Neatly organized	Attended to Precision	Correct

Resources: Knowledge is

Complete	Sound with minor errors	Some but significant faults appear	No knowledge

Beliefs and Attitudes: Problem Solver Exhibited

Persistence	Confidence	Curiosity

Geiger, V., & Galbraith, P. (1998). Developing a diagnostic framework for evaluating student approaches to applied mathematics. *International Journal of Mathematics, Education, Science, and Technology*, 29. 533-559.

APPENDIX 2 Julia's Camera-Ready Classroom Poster

Is my work Camera Ready?

Perseverance, solution pathway(s)… ACTION!

- Have I underlined the givens, circled the goals, and made a reasonable conjecture?
- Do I have a labeled picture, model, or diagram?
- Are my calculations and work precise with:
  - * units labeled?
  - * vocabulary that relates to the problem?
  - * an answer written in a complete sentence?
- How can others construct meaning from the work I've shown?
- How might I work connect to other ideas?
- How my work could be used as a collaboration tool to improve my craft?

(APPENDIX 3) Original Problem: Underline the givens and circle the goals.

Name

Date Hour

Conjecture (reasonable guess):

Plan: What will you do to solve this problem?
(I am going to … so that I can …)

Questions for getting unstuck:

Have I tried the strategies and followed my plan?

Am I getting closer to solution?

Can I break this problem into smaller pieces?

Other questions I asked myself:

Strategy Selection:

1. Draw and label a diagram.

2. Examine special cases.

3. Simplify the problem.

4. Consider equivalent problems.

5. Consider slightly modified goals and subgoals.

6. Act out the problem/ use manipulatives.

7. Identify what does NOT work.

8. Work backwards.
(Adapted from Schoenfeld (1998))

Solution Pathway (Not just what you did, but WHY you did it. Remember, your work is for everyone!)

Answer (Stated in a complete sentence, referring to the question, units included as necessary):

Check (verification): Use mathematical reasoning to prove that your answer is correct.

I know my answer is right because…

Underline the givens and circle the goals (1)	Conjecture and plan are appropriate and make sense in context of the problem (3)	Solution pathway is complete; questions are present; solution is checked for reasonableness and verified. (4)	Answer is correct, and is stated in a way that responds directly to the question asked.(2)	Total (10):

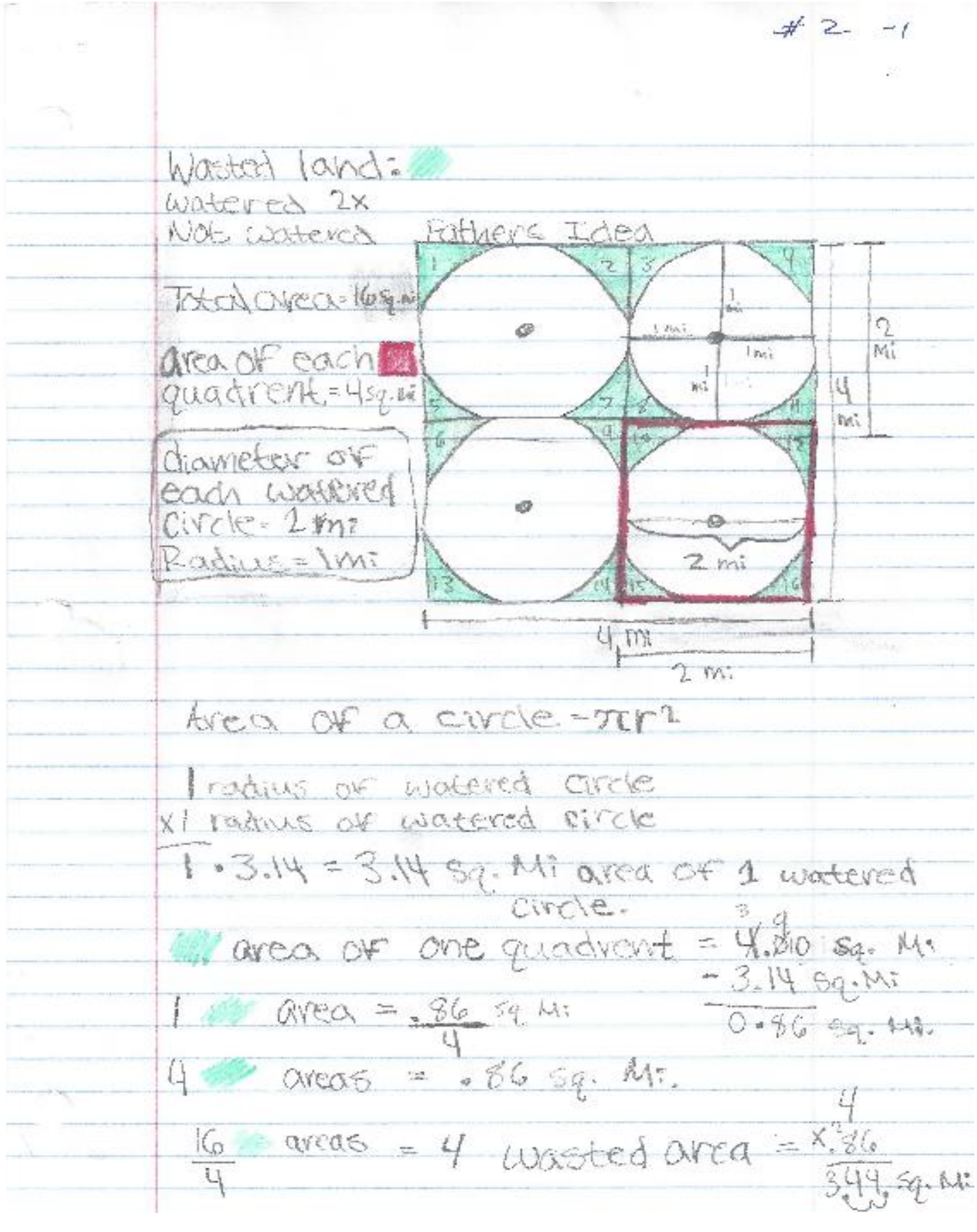
Template: (Perales, M. and Vicich, J., 2014)

APPENDIX 4 Portfolio Excerpt: Sample of Student Work and Teacher's Reflection

Problem: A farmer has a square plot of land measuring 16 square miles. He wants to irrigate his crops in a circular pattern with one system that will be moved around. Any land that is double-watered is considered unusable for farming. The farmer has been irrigating his land as four circular fields for many years. His son thinks it may be more efficient to add a fifth irrigation site at the center of his land. Which irrigation plan has the least "wasted" farm land?

Excerpts from Susan's analysis of (an average sample) Student #2's work throughout the year:

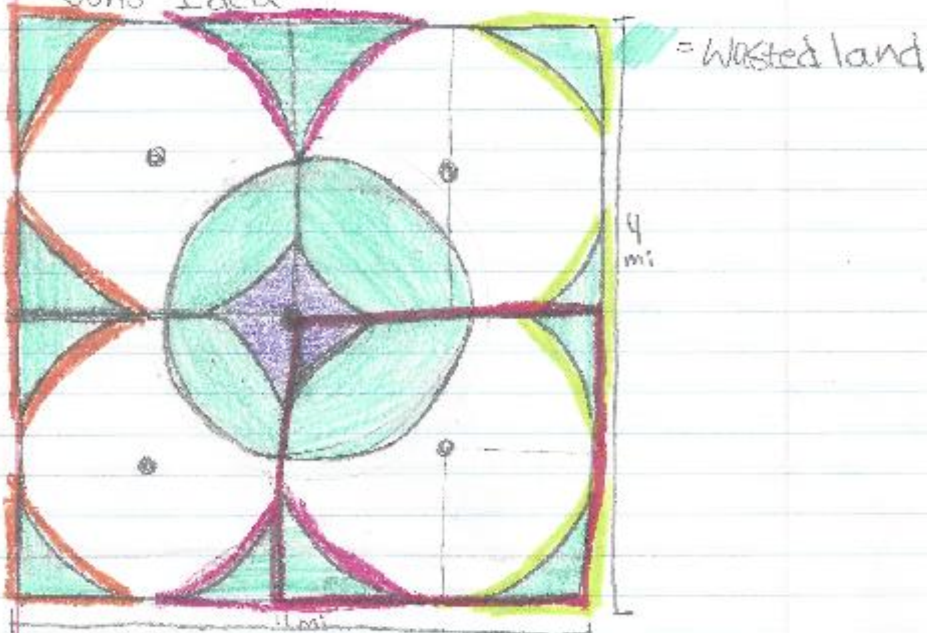
"I was pleasantly surprised at Student #2's use of color in her solution … it is evident this student utilized coloring as a tool to help her understand, organize and solve the problem. Additionally, the fact that her thinking centers on visualization of real-world situations and how they translate to math concepts was an advantage for her with this problem and she was able to come up with this plan more quickly than her peers. Student #2 has come a long way this year with her confidence in her own unique thinking, She has learned that it is perfectly acceptable for her to solve math problems using the strategies she is comfortable with instead of only [the] one way the teacher expects everyone to master. She is starting to realize that that some methods and strategies are more efficient than hers."



2 -1
Wasted land:
watered 2x
Not watered
Fathers Idea
Total area = 16 sq. mi
area of each quadrent = 4 sq. mi
diameter of each watered circle = 2 mi
Radius = 1 mi
2 Mi
4 mi
2 mi
4 mi
2 mi
Area of a circle = πr²
1 radius of watered circle
x1 radius of watered circle
1 • 3.14 = 3.14 sq. Mi area of 1 watered circle.
area of one quadrant = 4.00 sq. Mi
− 3.14 sq. Mi
0.86 sq. Mi.
1 area = .86/4 sq. Mi
4 areas = .86 sq. Mi.
16/4 areas = 4 wasted area = 4 x .86
3.44 sq. Mi

#2 -2

Sons Idea



total area = 16 sq. Mi.
area of each quadrent = 4 sq. mi
area = 4 blue corners / .86 sq Mi.
1 Circle = 3.14 sq. Mi.

3.14 sq Mi. area of one circle
- .86 sq Mi. area of
2.28 Center wasted area

= .86 sq Mi.
= .86 sq Mi.
= .86 sq Mi.

.86
x 3
2.58

2.28 sq. Mi. / Center wasted Area
+2.58
4.86 wasted land
Sq. Mi.

4.86 Sq. Mi. wasted land

The Sons watering system wastes More land than the Fathers by 1.42 sq. Mi. There For the Father has a better watering Method.

$$\begin{array}{r} 4.86 \\ -3.44 \\ \hline 1.42 \end{array}$$

APPENDIX 5 Problem-Solving Portfolio Instructions

During the 2015-16 school year, you will compile a portfolio of student work demonstrating some of your students' problem-solving skills. This assignment is designed to provide you with the opportunity to analyze and reflect on the effects of your teaching practices relative to students' problem-solving behaviors/skills throughout the entire school year. Have fun with it, and help your students grow to love becoming proficient problem-solvers! **Your portfolio will be due at the 4th Saturday Workshop on April 2nd, 2016.**

Use the following checklist to ensure that your portfolio contains key items and information:

<table>
<tr><td>☐</td><td>By September 15, 2015 identify three students, one high-performing student, one student who would be considered of "average" ability for the grade level that you teach, and one student who is a low-performer.</td></tr>
<tr><td>☐</td><td>Throughout the year, collect three to five class activities or homework problems that focus on problem-solving skills.
• Make copies of each student's work for the problems that you select prior to grading and after you have graded the paper.
• Make sure that the work samples are dated, scored, and arranged in chronological order for each student.
• Your students may use a problem-solving template to organize their work, but a formal template or other document is NOT a requirement.
• The collection of student work should demonstrate growth of each student, if possible.
• Scoring rubric for each task or an explanation of how the tasks are graded.</td></tr>
<tr><td>☐</td><td>Type a one-page (max) Student Growth Summary & Analysis for each student.
• Attach the summary for each student on top of his/her work samples from the year.
• Consider students' progress relative to specific behaviors such as:
  o initial engagement
  o monitoring of progress
  o making plans for solution paths
  o making conjectures
  o strategy selection
  o verification and justification of work
• You may also consider changes in students' beliefs such as perseverance, willingness to explore alternative strategies, etc.</td></tr>
</table>