Curriculum Enactment as Professional Development

Five curriculum development projects completed by teachers participating in the *College Ready Math-Science Partnership* are presented here. These projects are intended to provide experiences that supplement and extend the *College Ready* workshops and activities of the Professional Learning Communities through intensive examination of the design of a single lesson or coordinated sequence of lessons. The participant could revise an approach currently used in their classroom or they could revise curriculum materials provided in workshops. In these revisions they applied instructional methods emphasized at workshops:

- How to shift toward student-directed, guided inquiry instruction
- How to apply learning cycles and evaluate instruction using RTOP
- How to begin with clear learning objectives to apply backwards design
- How to incorporate anticipated barriers and possible responses in the design

Participants arrived at objectives for their own learning through conversation with the project facilitator based on the questions:

- What materials did you choose and why?
- What learning objectives are supported by these materials?
- What is your role as a teacher in their use? What is the role of the student?
- What will the student bring to instruction and how will this be confirmed?
- What challenges will students need to confront in these materials and how will the presence of these challenges be revealed?
- What instructional sequences will create a scaffold from which students can successfully interact with these materials?

Participants were provided with a summary of research on learning cycles and inquiry-based instruction. Participants were provided with summaries of the results of surveys of *College Ready* teachers and students that measure readiness to learn. These survey items were taken from published research on the effect of beliefs about the nature of science and the student's role in learning. A thorough description of this approach to the assessment of student-learner characteristics will appear elsewhere[1].

As reported by participants, the time spent on each project was between twenty and thirty hours:

- two or three, one-hour telephone conversations that initiated the project (3 hours)
  - first call emphasized topic selection questions
  - second call emphasized student beliefs survey, learning objectives and learning cycle
  - facilitator's notes and audio recording of each shared with participant
- two to four email-based discussions that refined and outlined the project (2 hours)
  - the initial outline was generated by the facilitator based on telephone conversation
- two or three revision of drafts of learner characteristics and learning objectives (10 hours)
  - the initial draft was generated by the facilitator based on emails and initial telephone conversations
- revision of drafts of teacher notes and student materials (10 hours)
  - last revisions emphasize anticipated student challenges and assessment

Participants received $500 at the completion of the project.

[1] *Disposed and Indisposed to Inquiry*, J. Eggebrecht, D. Reed, and G. Stewart submitted to Phys. Rev, September 2012

An Overview of the Five Projects Presented

Don Murphy (Oklahoma School of Science and Mathematics, Poteau, Oklahoma)

Murphy's work was based on a data collection technique that he had developed earlier. His goal in developing this method of using retired phonograph turntables was to help students better understand rotational motion due to a frictional force. Discussion with the facilitator revealed other specific learning objectives for his students involving their ability to design experiments and analyze experimental data. In the past the technique had used with a step-wise procedure provided by the instructor for implementation and analysis. This approach was replaced with one in which the students developed the strategy in small-group and whole-class discussion.

Darrin Grainer (Greenbrier High School, Greenbrier, Arkansas)

Grainer revised an approach for the study of projectile motion that he was currently using that did not use microprocessor-based data acquisition. The idea was to provide a situation in which students could choose to use probe ware in order to reduce uncertainty in measurements leading to ambiguous conclusions. If with a stopwatch and ruler the model cannot be satisfactorily tested, the experiment could be redesigned by the student using instruments with improved precision. As a second purposeful application of technology, Grainer incorporated a TI-based programming activity for the prediction of trajectories based on the student model. Important learning objectives beyond a deeper understanding of the equations of motion included the ability to select methods of measurement and the ability to program the Texas Instruments calculator. The existing approach was also reconstructed within a 5Es learning cycle and evaluated with RTOP.

Suzanne Loucks (Pangburn High School, Pangburn, Arkansas)

Loucks' project was to redesign her lessons on constant velocity motion. One of her goals in this work was to support student achievement in mathematics. Her work documents her deep understanding of student characteristics and the challenges they encounter in the graphical representation of data. The Teacher's Notes capture successful interactions with her students that have been framed by a learning cycle.

Doug Reed (Pulaski High School, Little Rock, Arkansas)

Reed revised *College Ready* workshop materials to match his students and classroom. Reed expressed concern about the ability of students to succeed in inquiry-based instruction. This work was, in part, an exploration of the extent to which step-by-step instructions on the use of probe ware were needed. An MBL activity appropriate for a lesson very early in the school year was selected and a revision of a guided-inquiry approach from the *College Ready* workshops was implemented. The purposes of this shift to a more student-directed approach were examined in terms of the learning objectives that could be supported. A learning cycle, incorporating methods similar to the Modeling Approach, was used.

Johnny Turrant (Northside High School, Fort Smith, Arkansas)

Johnny revised *College Ready* workshop materials to make a capstone project-based activity that applies energy conservation, circular motion, and principles of engineering design. Several levels of student-direction in a project involving the design of a roller coaster were included in workshop materials provided to *College Ready* participants. This work used the most open-ended of these and revised the learning cycle to support this approach.

Measurement of the Coefficient of Static Friction

Don Murphy
Oklahoma School of Science and Mathematics, Regional Center
Poteau, Oklahoma

Overview

In analyzing mechanical systems students need to be able to identify the forces that are present. Students sometimes think of centripetal force as a force exerted on the object by the motion of the object rather than a force caused by gravitational or electromagnetic interactions. This lab activity addresses that problem.

Contact forces including tension, spring forces, and friction are secondary forces. They are useful representations in which the underlying electrostatic interactions aren't apparent and neither is the fact that these forces are due to a change in the net forces exerted on atoms. The idea that the tension in a rope *appears when needed* as you try to stretch the rope, however, is easier to grasp than the idea that a friction force *appears when needed because the object would otherwise slide*. And the direction of the force is whatever it needs to be to oppose the potential motion of the object. Like the perception that a centripetal force is caused by the motion, a frictional force might be perceived to be caused by the potential motion of the object. This lab activity addresses that problem.

This lab activity uses a turntable to provide an opportunity to investigate frictional forces exerted on an object in uniform circular motion. The activity can be used in an algebra-based course on mechanics or in a mechanics course that uses calculus. The amount of guidance needed by the students is different in the two situations. In both, the activity is integrated with PhET simulations, lectures, and other, less formal hands-on activities are used in an instructional sequence that includes uniform circular motion and frictional forces.

Student Characteristics

The Oklahoma School of Science and Mathematics Regional Center at Poteau provides opportunities for talented juniors and seniors to study calculus and physics. It is a publically funded, residential school. Students are highly motivated and admission is selective. It is expected that students will complete a year of calculus before graduation.

A one-semester mechanics course that assumes only algebra and trigonometry is offered in the first semester. A calculus-based mechanics course is offered as a one-semester course in the second semester. The textbook for the first semester course is *The Physics of Everyday Phenomena* by Griffith and the second semester course uses *Fundamentals of Physics* by Halliday, Resnick and Walker.

The emphasis in the algebra-based course is on conceptual understanding. During the first-semester course, students are still uncomfortable when they are not provided with explicit instructions. However, admission to the program is competitive and these students have the ability to read instructions and reason. During the second-semester course students are more willing to take risks and are able, with support, to develop an experiment for the system considered here. In both courses, classes typically with less than twelve students, meet each day for a single period.

In both courses students have covered Newton's second law and used force diagrams. Prior to the activity students in both courses have been introduced to the relationships between frictional force and normal force. A mathematical treatment of angular motion is not included in the first-semester course. However, a conceptual treatment includes the idea that linear motion in a circular orbit is tangent to the path and increases with the radius at constant angular frequency. Also, the idea that centripetal acceleration decreases with radius and increases with the square of the linear velocity is included. In the second-semester course, students use mathematical representations of these relationships. They have also studied rotational variables and can express the velocity and acceleration in either linear or angular variables.

Prior Knowledge
In order for students to be able to succeed they need to be able to:

- Describe the dependence of centripetal acceleration and linear velocity on angular velocity and radius qualitatively.
- Explain the relationship between centripetal acceleration and frictional force in terms of Newton's second law.
- Describe and explain the behavior of the frictional force on an object in uniform circular motion as the angular velocity and/or radius are varied until the object slips.
- Draw a force diagram for an object in uniform circular motion with correct representations of direction and the qualitative dependence of the magnitude on angular velocity and radius.
- Identify the contact, gravitational or electrostatic forces that are exerted as a centripetal force.

Learning Objectives
Lab skills are often lacking for these students. This is the real value of the activity. It provides an opportunity for measurement with precision, recording and organizing data, and to be able to summarize their observations and results in writing. Beyond a deeper understanding of frictional and "fictional" forces, the goals are for them to be able to:

- Make and record accurate measurements of angular velocity and radius, including precision
- Make and record accurate measurements of radius at which an object in circular motion slips, including the use of replicas to estimate precision
- Make and defend predictions of the static friction coefficient for pairs of surfaces not investigated by the class

Instructional Strategies
A procedure could be provided. However, the goals of the activity are better served if students develop the procedure. For students in the algebra-based course it is necessary to develop the procedure with them during whole-class discussion. With the calculus-based class students can develop the procedure in small teams, although this practice

requires an additional instructional period. As described in the handout for students they are required to discuss their design with the instructor before beginning the experiment.

Either through whole-class discussion or with questioning in small groups, students can be guided toward the following sequence:

1. Record the mass of each disc (steel and ceramic discs are provided).
2. Cut the paper sheet into a circle approximately the same size as the turntable of the record player (several turntables are provided).
3. Punch a small hole with a pencil at the circle's center.
4. Stick a piece of masking tape on the paper extending from the circle's center to its edge.
5. Carefully mark the tape (from center towards edge) in centimeters and millimeters.
6. Repeat steps 1 through 4 for the plastic sheet.
7. Place paper sheet over pin in center of turntable.
8. Place a disc near the center of the paper sheet.
9. Turn on the turntable at slow speed.
10. If the disc does not spin off the sheet, stop turntable, move disc further towards edge and start again.
11. Repeat until you find the point at which the disc spins off the sheet.
12. If disc does not spin off the sheet, switch to higher speed and repeat steps 6 through 10.
13. Record the turntable speed and accurately read the distance from the center of the last trial in which the disc did NOT spin off the sheet. From these you will determine the centripetal force on the disc when static friction was at its maximum value.

Students must also present their strategy for data analysis. And two preliminary runs of the experiment are required before finalizing and presenting the plan. Either in whole-class discussion or by questioning small groups the strategy for the analysis of the data should be based on the identification of the input and output variables and be connected to relationships among them that have been developed in the preceding instructional sequence:

Disc material	Disc mass (m)	Surface material	Radius at which disc slipped (r)	Velocity at which disc slipped (v)	Centripetal force (F_c)	Maximum force of static friction (f_{smax})	Coefficient of static friction (μ_s)
Steel		Paper					
Steel		Plastic					
Ceramic		Paper					
Ceramic		Plastic					

Anticipated Challenges

My assumptions of what they know about the relationships among these variables are sometimes not correct. And when I visit the lab groups I have an opportunity to review these concepts when a need for a review is evident:

- When an object travels in a circular path the direction of the object's velocity changes from moment to moment. This is centripetal acceleration.

$$a_c = v^2 / r$$

- This acceleration is caused by a force or forces that fill the role of 'centripetal force'. There is no force in nature that is strictly known as 'centripetal' force. This role must be filled by a 'real' force or a combination of real forces such as gravity, normal force, tension and friction.
- We calculate the centripetal force in any situation using a form of Newton's 2nd Law:

$$F_C = ma_c$$

- Static friction can be the force that provides this acceleration

$$F_C = f_{s\max}$$

- Remember, static friction can range from zero to some maximum in response to efforts to cause an object to slide on a surface.

$$f_s \le \mu N \quad \text{and} \quad f_{s\,max} = \mu N$$

Also, when visiting the lab group either to review their experimental design or plan for the analysis of data, problems are expected. The presence of these problems can be indicated by a cue. Possible responses are "in the back pocket."

Cue: They are not always careful about finding ***<u>the</u>*** point at which the discs start to slide.

Response: I might ask how the outcome would change if the disc was just a little bit further from the center, if there are any patterns in the angle through which the turntable rotates before sliding off, if they are sure that the turntable is rotating at constant velocity when the disc slides off, or if they are always measuring the location of the disc in the same way.

Cue: Sometimes the discs slide at a slightly different point over several consecutive tries.

Response: I might suggest that they look to see if there is a pattern that needs to be explained. If there is not, then I ask if their data table has a column for each trial and a column for the average of the trials. If their planned experimental design has included a decision about the number of trials, then I might ask how they know if this number of trials is enough. And if they haven't included a decision about the number of trials, then I can ask how we missed that.

Cue: I have had students jump to the conclusion that the radius of the turntable surface is important, not the radius of the path at which the object starts to slide.

Response: We go back and review the behavior of the system talking through each step. Placing the disc far from the center we see an immediate "effect" and consider "what could change so that this didn't happen?"

Cue: Some think that as the position of the disc gets further from center, less friction force should be required, since r is in the denominator of the centripetal acceleration. They fail to consider that as r increases, the velocity increases as the square of r, requiring more friction force as r increases.

Response: We can look at the numerator and denominator separately. I can ask them to tell me the unit of the turntable rotation speed. I can ask them to give me the unit of the velocity. I can ask them to tell me about the relationship between these. I can ask them to do the algebra needed to write the centripetal acceleration as a function of r multiplied by constants.

Measurement of the Coefficient of Static Friction

OBJECTIVES

- Devise a method to determine the coefficient of static friction between materials.
- Gather data involved in circular motion
- Calculate the centripetal force.

MATERIALS

- Turntable
- Sheets of different composition
- Discs of different composition
- Masking tape
- Meter sticks
- A balance

PROCEDURES

You know about friction and circular motion. Given these materials, devise a method for determining the coefficients of static friction between the discs and the sheets.

Write down each step in your procedure. Make sure that your description identifies the independent and dependent variables, their units, the range and number of values that will be used for the independent variable, and the way in which the dependent variable is to be measured. If the coefficient of static friction is not measured directly then describe how you will calculate it from the data that you collect.

Make a sketch of the system and in the sketch show the variables that you are measuring and a scale that shows increments for the independent variable. Add notes to the drawing that describe your predictions of how the system will behave.

When you have a sketch and a plan written that each member of the team agrees with, ask your teacher to discuss these with you.

DATA ANALYSIS

Before you start to collect data prepare a way of organizing and recording the data. Perform a trial run with your procedure to see if your organization of the data makes sense.

Calculate a preliminary value of the coefficient of static friction from this first data. Based on the measured values that were used in this calculation estimate the precision of your value of the coefficient. Make another run to see if the results produce a value of the coefficient that agrees with your first value. Revise your procedure and your scheme for recording the data if necessary. Discuss your plan for the analysis of your data and your preliminary results with the teacher.

SUMMARY

Describe your method and show how you gathered data and what calculations you made to arrive at your conclusions. Include in your description:

- calculations of the centripetal force for two different values of the independent variable from your data
- a diagram showing the direction and magnitude of the forces for these two cases
- a sketch of the frictional force for several values of the independent variable

Make sketches of these systems. In the sketch show the direction of the centripetal force and label the interactions that are responsible for the centripetal forces in these systems:

- a planet orbiting a star
- a car on a Ferris wheel or roller coaster
- an electron orbiting a positive charge
- a passenger in a car moving along a curve in the road

Compare coefficients of static friction for all pairs of materials that you studied. Describe conclusions drawn about the coefficients and properties of the surfaces. Based on these observations predict the relative sizes of frictional forces for pairs of surfaces that were not examined in this experiment and justify your prediction.

Predicting a Trajectory

Darrin Grainer
Greenbrier High School
Greenbrier, Arkansas

Overview

This activity involves predicting the trajectory of a ball launched from a track and designing an experiment to test the prediction. This activity would apply to regular, Pre-AP or honors Physics, and AP Physics. Classes in these courses at Greenbrier High School in Greenbrier Arkansas meet each day for 50 minutes and have 12-20 students. This project involves experimental design and so will span more than a single day. After doing the activity the students will acquire a better understanding of two-dimensional motion. This activity allows for a low-tech approach without the use of probeware as well as a follow-up method that allows the students to use probeware to collect the data. This will be used to teach students that the choice of appropriate equipment can greatly increase the precision of the experiment.

Materials and Preparation

The data collection for this activity can be done using a stop watch, flexible track from Lowe's, meter sticks or metric tape measures, steel ball bearings, Vernier photogate timers interfaced with a LabQuest or Labpro system, and a ring stand with universal clamp.

Relevant Arkansas Science Standards

MF.2.P.6 Describe the path of a projectile as a *parabola*

MF.2.P.7 Apply *kinematic* equations to solve problems involving projectile motion of an object launched at an angle:

$$v_x = v_i \cos\theta = \text{constant}$$

$$\Delta x = v_i(\cos\theta)\Delta t$$

$$v_{y,f} = v_i(\sin\theta) - g\Delta t$$

$$v_{y,f}^2 = v_i^2(\sin\theta)^2 - 2g\Delta y$$

$$\Delta y = v_i(\sin\theta)\Delta t - \tfrac{1}{2}g(\Delta t)^2$$

NS.17.P.1 Develop the appropriate procedures using controls and variables (dependent and independent) in scientific experimentation

NS.17.P.4 Gather and analyze data using appropriate summary statistics (e.g., percent yield, percent error)

NS.17.P.5 Formulate valid conclusions without bias

NS.19.P.1 Use appropriate equipment and technology as tools for solving problems (e.g., balances, scales, calculators, probes, glassware, burners, computer software and hardware)

NS.19.P.2 Manipulate scientific data using appropriate mathematical calculations, charts, tables, and graphs

Student Learning Objectives

In addition to providing a deeper understanding of projectile motion this activity has these goals:

- Students will be able to write down a procedure for a lab without having been given all the steps.
- Students will learn to work together collaboratively.
- Students will gain a greater understanding of two-dimensional motion.
- Students will learn to select and use data collection probeware to improve the precision of their measurements.
- Students will use TI calculators to analyze the data collected.

The Learning Cycle

This activity uses the 5Es learning cycle. Students are shown the physical situation; a ball flies off of the end of a horizontal track and lands some distance away. As a class they discuss the situation and build a model that describes it. In small groups they then test their models by predicting the distance that the ball travels. Students return to a whole-class discussion where each group reports on the results of their tests. Either a consensus model is developed or the models and methods of measurement of initial velocity are revised and the tests repeated. Finally, to strengthen understanding by developing a new representation, students are asked to write a program to calculate the landing site from the height of the launch.

Throughout this activity white boards are employed by the students to explain their experimental design and results of their experiment. They are used to support guiding questions from the teacher when visiting the lab teams. And they are used by the team to report results.

The activity is designed to score well on the Reformed Teaching Observational Protocol (RTOP), since the RTOP has been shown to identify instructional strategies that improve student learning. In each of the 5 phases of the instructional design the connection to the RTOP is described.

Engagement

The students are shown the phenomenon (a ball descends a track, flies off and then lands). This is followed by a whole-class discussion to brainstorm the variables (they can find "how high", "how fast", "how far", and "how much time in the air"). It is useful to guide the selection of variables (for example, with "what equipment do we have?" and "how can we measure these?") so that we avoid assumptions of a frictionless surface and the application of energy conservation with a rotational kinetic energy term. Also, the teacher may want to guide students towards the initial use of the low-tech method that is assumed in the following description. If students or the teacher elect to use probeware the students will still be able to design the experiment using equipment provided.

The whole-class discussion is used to guide the identification of a dependent and independent variable (how far and how fast) and to guide the development of a model (for example, "the distance is proportional to the initial speed"). However, the question of how to describe the constant of proportionality will remain to be found through this discussion. For this purpose, questions can be asked that prompt the students to identify the height of the table as an important factor. In order to complete the model, dimensional analysis can be used to see that the constant of proportionality has units of time. A complete model requires that the students identify this time as the time it takes the ball to fall vertically from the launch height.

A final model that can be a goal of this discussion is:
The horizontal distance traveled by the ball is proportional to the product of the horizontal velocity of the ball when it is launched and the time that the ball is in the air.

The time that the ball is in the air is the time that it takes the ball to fall from the launch height.

Many students will have trouble going from the written statement of the model to the mathematical statement. Support them by writing the statement on the board and then underlining the variables. Ask them to complete the sentence, "Let x be …" When they have a sentence for x record it on the board. Do this until each of the variables has its own definition and then ask the students to translate the written statement of the model into the equivalent mathematical statement by using substitution.

Purpose for the Engagement Phase
The traditional approach in which a teacher presents a concept and then the student performs a lab activity to confirm the concept is not always successful for my students. Observations using the Reformed Teaching Observation Protocol suggests that learning may be improved if the concept is introduced to the students through their observation of the phenomenon and a discussion of their ideas about what is important. In the RTOP table this strategy is awarded the highest value: "In this lesson, student exploration preceded formal presentation." Many other RTOP factors are present here. For example, if the discussion is conducted as described, then it is the student's conceptions that are leading and so "Students were involved in the communication of their ideas to others using a variety of means and media."

Exploration
Students develop a laboratory procedure that can be used to test their model and a strategy to analyze the results of the experiment so that a conclusion can be drawn regarding the correctness of the predictions made by the model. A whiteboard is used by each group to record the experimental design. The teacher visits each group and checks to see that the design is safe and productive. If there are flaws in the design, such as the neglecting to measure one of the variables, then questions are posed. However, as described below in the evaluation phase, groups will be able to redesign the lab if it fails.

A traditional procedure that is familiar to most teachers is likely to be adopted by the group: The ball is released from a height that is recorded. The velocity of the ball as it leaves the track is obtained by measuring the time to traverse a measured distance ending at the launch position. A careful procedure will investigate the reproducibility of this method of determining the velocity, such as variations due to distance along the track and estimates of response time biases. The horizontal distance from the launch site is predicted and measured. Students may decide to measure a diagonal path from the launch site to the landing site. A careful procedure will investigate the precision with which the landing site can be identified; students might think of using carbon paper or sand to improve reproducibility, particularly if they are available in plain sight.

It is important for the students to use the experiment as a test of the predicted value of the range based on their model. If, when visiting the team, the teacher discovers that this is not written down as an explicit step in the design, the group could be asked, "Why are you doing these steps? What is this experiment being used to test?" It is also important that the students are including replicas in their design. Otherwise, they will be confronted with a comparison of two points: the predicted value of the range and the point taken from a single measurement. If, when visiting the team, the teacher discovers that there are not attempts made to determine the interval of distance in which the experimental result lies, then the teacher has an opportunity to have a conversation with the students about measurement. If might begin with, "Will the ball always land at precisely that spot?"

Purpose for the Exploration Phase
Every laboratory can't involve students in experimental design. However, this lab will work well to enhance learning by supporting all of these RTOP factors:

- Students made predictions, estimations and/or hypotheses and devised means for testing them.
- This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.
- The focus and direction of the lesson was often determined by ideas originating with students.

Explanation
When the exploration phase is complete, students need to perform simple statistics in order to determine whether or not the predicted value of the range fell in the interval observed in repeated measurements. A return to whole-class discussion and reporting out of the results may indicate that the predictions have consistently been outside of the experimental interval. In this low-tech approach response time in the measurement of the time interval along the horizontal track can lead to a measurement of initial velocity that is biased toward velocities that are smaller than the true value, since the measured time is likely to be larger than the real time. Also, frictional losses occur along the horizontal track, so an average velocity that is measured will be larger than the instantaneous velocity at the end of the track. As the distance along the track used in the measurement decreases, the average gets closer to the true launch velocity. However, the difficulty of making measurements of the time interval increases. The whole-class discussion will allow an opportunity to thoughtfully investigate these effects and, by only using more precise methods of measuring the launch velocity, this opportunity is lost. When a need for greater precision is identified in order to test the model, the use of photogate timers can be introduced as a solution to an authentic problem.

A method for refined velocity measurements using probe ware is to mount a pair of photogate timers along the path beyond the launch point. A description of this approach is available from Pasco at http://www.pasco.com/physhigh/force-and-motion/projectile-motion.cfm). Before allowing the redesign of the procedure using improved measurements, the discussion should proceed far enough so that groups whose approach had been non-productive will have a chance to recover.

Purpose of the Explanation Phase
The use of photogates for the measurement of the launch velocity should depend on whether or not the issues raised by the students warrant the use of a more precise measurement. The learning objectives will have been served if the students have developed a model and tested it using the available experimental data.

Allowing them to make the decision serves these RTOP factors that enhance learning:

- There was a high proportion of student talk and a significant amount of it occurred between and among students.
- Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.
- Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.

Elaboration
Students will use the TI-84 graphing calculator along with the results of their experiment to develop a program to calculate the range of a projectile given different variables such as ball diameter and height of lab table.

Guidance on the Program
Unless students program the TI calculator frequently, they will need a quick refresher. They probably will not have a problem with inserting the necessary operation (*, /, and $\sqrt{\ }$) into the program editing window. But the input and output functions may be forgotten. So some guidance is provided by a set of notes taken from the User's Guide.

The program involves only a few steps. The teacher can visit groups and ask questions that guide the team to include each step in the following sequence:

Input h	:PROMPT h
Calculate t	:t= $\sqrt{\ }$ (2*h/9.81)
Input v	:PROMPT v
Calculate x	:x=v*t
Output x	:DISPLAY(x)

Purpose of the Elaboration Phase
Just as translating between graphical and mathematical representations of concepts deepens understanding, so will programming the two-dimension equations of motion on a hand-held calculator. Here the student is applying these equations in a new situation (obtained by varying a parameter).

Among the relevant RTOP factors are these:

- The teacher acted as a resource person, working to support and enhance student investigations.
- Students were actively engaged in a thought-provoking activity that often involved the critical assessment of procedures.

Evaluation

The evaluation will be done in the form of a free-response question given on the unit test. The question will involve identifying the appropriate equipment needed to determine the range of a projectile as well as writing a procedure for accomplishing this task. They will be asked questions like, “What will happen to the range if the table is raised assuming the same initial horizontal velocity?” or “Why was it not necessary to measure the initial drop height from the top of the track?” or “Why is it important to determine the horizontal velocity of the ball as close to the edge of the table as possible?

Alternative Evaluation:

The teacher prepares an event in which a variable (in this case the initial velocity and height of launch point, since the TI calculator has been programmed) are unknown until the students enter the classroom. Also, students could be given a new ball with an unknown and different diameter, if a single photogate is used where the time interval during which the beam is blocked will be measured. Depending on the nature of the class a race could be held, or a time limit could be set, for students to predict, test, analyze, and summarize their results.

Predicting a Trajectory
Student Handout

During our discussion you identified the variables needed to describe and predict the range of a projectile launched from the tabletop. Discuss within your group your individual models of this behavior. When you have agreed upon the model or models that are to be tested record them on your white board using both a mathematical expression and a written statement.

After you've completed your test of the predicted value of the range you will need to report your results to the class. Choose a member of your group to present your work.

Be sure to include the following Information:

I. A sketch of your model
II. A short step-by-step procedure
III. Answers to the following questions using your model
- What is the independent and dependent variable?
- In what units are these variables expressed?
- How will you estimate the precision of measurement of the independent and dependent variables?
- Are there alternative ways of measuring the independent variable? If so what are they?

Working together as a group, develop a strategy to analyze the data that will be produced by your procedure. Answering the following questions may help guide your strategy.

- Are there portions of the range of the independent variable where data is unreliable and, if so, what is the reason?
- Are there unknown constants that might result from the analysis whose comparison with accepted values would support or refute your model?
- Would changing variables such as the mass of the ball bearing, diameter of the ball bearing, or height of the table affect the range of your projectile?

A New Representation of Your Model

Working with your team, develop a program for your TI calculator that uses your model to predict a value of the range of the ball.

The following notes from the User's Guide may be useful:

Input [*variable*]

You can display *text* or the contents of **Str***n* (a string variable) of up to 16 characters as a prompt. During program execution, enter a value after the prompt and then press [ENTER]. The value is stored to *variable*, and the program resumes execution.

Input ["*text*",*variable*]
Input [**Str***n*,*variable*]

Program

```
PROGRAM:HINPUT
:Input A
:Input L1
:Input "Y1=",Y1
:Input "DATA=",L
DATA
:Disp Y1(A)
:Disp Y1(L1)
:Disp Y1(LDATA)
```

Output

```
prgmHINPUT
?2
```

```
?{1,2,3}
Y1="2X+2"
DATA={4,5,6}
              6
        {4 6 8}
     {10 12 14}
           Done
```

Output(*row*,*column*,"*text*"**)**
Output(*row*,*column*,*value***)**

Program

```
PROGRAM:OUTPUT
:3+5→B
:ClrHome
:Output(5,4,"ANS
WER:"
:Output(5,12,B)
```

Output

```
ANSWER: 8
```

Test your calculator model by varying any of the above variables to see if it fails. If it does, explain why, and then debug it.

You will need this program in the future, so save it with a name that you will remember.

Constant Velocity

Suzanne Louks
Pangburn High School

The Setting

The project described here will be used with either Physics or Physical Science students at Pangburn High School in Pangburn, Arkansas. These classes meet for a single period five days a week. There are usually about twenty students in the Physics class and about thirty students in the Physical Science class.

These materials provide an alternative to the probeware investigation of constant velocity motion used in the *College Ready* workshop. The materials have been used with Physical Science students early in their study of kinematics at the beginning of the second semester and will be used as a review of constant velocity for Physics students early in the first semester. For either class it is natural to follow this work with the concept of constant acceleration.

For the Physical Science students these materials are intended to provide support for their development of skills that are emphasized in their algebra 1 course and identified in the *Arkansas State Standards*, as described below in the section on learning objectives. Unlike science, skills and conceptual understanding in mathematics are tested and the stakes are high both for the students and the school. These materials have been discussed with mathematics teachers who have shown interest.

For Physical Science students these materials use a structured inquiry. Physical Science students require more explicit direction than do Physics students. With greater maturity comes an increasing ability to assume responsibility for your own work. And so, in these materials the Physical Science students are provided with an outline of the procedure to be used for data collection and a form to be used as they collect data in their lab notebooks. For the purposes of reviewing essential content and to increase their ability to be self-directed, the Physics students are asked only to tackle the more difficult part of the task; define a sequence of steps that can be used by a robot to automatically calculate speed and velocity. For Physics students, these supports are not needed and a guided inquiry is used.

Students are asked to collaborate with each other to complete these tasks and white boards are used to support the collaboration. For the Physical Science students, the ability to collaborate is an explicit expectation in the learning objectives and so this appears in the rubric used to evaluate their work.

For Physical Science students, the activity on the second day is best done outside on pavement or in the gym. These students tend to get excited and want to play with the cars; giving them a time limit seems to help. After taking data, students go back to the classroom and work together to finish the lab. When students have completed graphs, you may want them presented to the whole class for discussion. This makes a great white board activity and progress toward the Physical Sciences standard on communication is assessed in the rubric.

For Physics students to have an opportunity to review concepts and skills acquired earlier, it is sufficient to provide them with an opportunity to collaboratively design the data collection and data analysis strategy and to then discuss their work presented on a white board. Illustrative data can then be provided to which they can apply their strategy for analysis. The Physical Science students will often find that their lines do not have a consistent slope. This is a great time to discuss variables, human error and reaction times (such as calling time and marking). For Physics students, the ability to analyze noisy data is an extremely important skill and a major part of their task is to develop a strategy to deal with real world data. Also, sometimes the cars turn instead of going straight and Physical Science students can be asked how to improve the data collection.

Science Standards and Learning Objectives

The *Arkansas State Standards* include these expectations for skills in mathematics that are supported by these materials:

LF.3.AI.4	Identify *independent variables* and *dependent variables* in various representational modes: words, symbols, and/or graphs
LF.3.AI.5	Interpret the rate of change/*slope* and intercepts within the context of everyday life (Ex. telephone charges based on base rate (*y-intercept*) plus rate per minute (slope))
LF.3.AI.6	Calculate the slope given • two points • the graph of a line • the equation of a line
LF.3.AI.7	Determine by using slope whether a pair of lines are parallel, perpendicular, or neither
LF.3.AI.8	Write an equation in slope-intercept, point-slope, and standard forms given • two points • a point and y-intercept • y-intercept and x-intercept • a point and slope • a table of data
LF.3.Al.9	Describe the effects of parameter changes, slope and /or y-intercept, on graphs of linear functions and vice versa.
NLF.4.AI.5	Communicate real world problems graphically, algebraically, numerically and verbally
P.6.PS.1	Analyze how *force* affects *motion:* one-dimensional (linear) two-dimensional (*projectile* and *rotational*)
P.6.PS.2	Explain how *motion* is relative to a *reference point*
P.6.PS.3	Compare and contrast among *speed*, *velocity* and *acceleration*
P.6.PS.4	Solve problems using the formulas for *speed* and *acceleration:* • $v = \frac{d}{t}$ where v = speed (velocity), t = time and d = distance
P.6.PS.5	Interpret graphs related to *motion:* distance versus time (d-t) *velocity* versus time (v-t) *acceleration* versus time (a-t)

The *Arkansas State Standards* include these expectations of physical sciences that are supported by these materials:

Physics students should have acquired the mathematical skills defined above as well as the Physical Science concepts, since that course is a prerequisite for enrollment in Physics. However, it often happens that Physics students can benefit from a review.

This activity helps students to achieve many of these Standards in mathematics and science by:

- identifying independent and dependent variables and distinguishing their roles in a mathematical relationship between position and time
- interpreting the slope of a position-time relationship as the rate of change called the velocity and interpreting the intercept as the initial position
- determining the velocity from position-time data and writing an equation in the slope-intercept form for the relationship between position and time
- explaining a physical situation in which motion is relative to a reference point and expressing that situation in a graphical representation
- solving problems involving velocity , time, and position
- contrasting speed and velocity
- interpreting and constructing motion graphs and
- communicating their thinking

The Learning Cycle

Often in mathematics and science, and in mathematics and science classrooms, the context for inquiry is a problem that must be solved. In this work the students develop the description of constant velocity motion over two days. In the first day small group work and guided, whole-class discussion is used to solve a series of problems that successively introduce aspects of constant velocity motion: speed, velocity, rate and slope, reference position and intercept, and situations in which different constant velocity motions are combined. On the second, day students collect and analyze data involving objects in motion.

This work is relevant in both mathematics and science instruction. Math teachers might see in the sequence of activities the learning cycle used in Cognitively Guided Instruction. More familiar to science teachers is the language of the 5Es (Engage, Explore, Explain, Elaborate, and Evaluate) learning cycle and that language will be used to describe this work. Language is used to organize ideas and construct meaning. The purpose of either of these descriptions is to organize instruction to improve learning. The research that led to the development of the Reformed Teaching Observational Protocol (RTOP) identified instructional strategies that improve student learning. In each of the 5 phases of the instructional design the connection to the RTOP is described.

Engagement Phase

We begin by making up scenarios that involve the child in a role in terms of their movement in the classroom or between places in town. With ninth grade students it is important to model the thinking processes involved in solving problems. Within a particular scenario the students are asked, what is your reference point? How do you know you have moved? And it is useful to put these guiding questions in a larger context such as, at night how can you tell if one light in the sky has moved?

Then speed is discussed. Again, personally engaging situations are more effective. Who is the fastest kid in the class, on the ball team, or the track team? How can you determine who is the fastest kid in class? What two properties help us determine speed? What are the dimensions of speed? The thinking required to build a model of motion can be pursued with questions. How do the speeds of two runners compare if one goes the same distance in a shorter time? How do the speeds compare if one goes a greater distance in the same time?

They compare and contrast speed and velocity during a discussion based on questions such as these:

If I told you to go find Greg who ran out of here at 10 meters per second, would you know where to start looking?

If I told you to go find Greg who ran out of here at 10 meters per second toward the Cafeteria, would you know where to start looking?

At this point the idea of a vector can be introduced: "the additional information in the second question is a direction that can be represented as an arrow and we call that a vector."

Purpose for the Engagement Phase

The contexts used are easy for the student to imagine. The relevant features of motion are discussed in the student's informal language and in a way that students actively participate as members of a learning community. In the RTOP rubric this is one of the most important elements of good instruction. Other factors that can be supported during this phase are important for sustaining student involvement in this work:

- In this lesson, student exploration preceded formal presentation.
- The instructional strategies and activities respected students' prior knowledge and the preconceptions inherent therein.
- There was a high proportion of student talk and a significant amount of it occurred between and among students.
- Active participation of students was encouraged and valued.
- The metaphor "teacher as listener" was very characteristic of this classroom.

Exploration Phase

During this session that prepares for the activity it is also useful to look at stories that help students make meaning of slopes, intercepts and graphical representations of constant velocity motion. Research-based approaches to these concepts in mathematics instructions allow students to *develop these concepts through problem solving*, rather than as an application of concepts developed by the teacher through presentation.

The following story provides an example. Students are asked to collaboratively develop a graphical representation of this story on a white board.

> Tim challenged Suzy to a race. Tim told Suzy she could have a 10-meter head start. The race is to the stop sign 100 meters away. Tim reaches the stop sign in 8 seconds. Suzy reaches the stop sign in 7.5 seconds. Suzy won the race but was she the fastest?

They are asked to add annotation:

- Where in your graph is the "head start?"
- How does the graph represent "100 meters away?"
- Where on the graph is Tim at 4 seconds? Where is Suzy at 4 seconds?

• How does your graph show that Suzy arrived first at the stop sign?
• How does your graph answer the question of who is fastest?

As the students work in small groups, the teacher monitors their progress. It is very possible that a group will have represented distance on the x-axis. Since one of the learning objectives is to be able to distinguish dependent and independent variables, it is useful to not intervene. During whole-class discussion following the small group work, questions such as “during the race would wrist watches on Tom and Suzy show the same time?”, “would what the time is for Tom depend on where Suzy is?”, “if position is the independent variable, is this Tom’s position or Suzy’s?”

Purpose of the Exploration Phase
The ability to translate between different representations is critical. The best way to acquire that ability is to make it make sense to you personally. Sharing your thinking with others as you do this helps you to clarify and refine what you are thinking. A host of factors identified in the RTOP rubric are present:

- Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.
- Students made predictions, estimations and/or hypotheses and devised means for testing them.
- Students used a variety of means (models, drawings, graphs, concrete materials, manipulatives, etc.) to represent phenomena.
- Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.
- Students were reflective about their learning.
- Intellectual rigor, constructive criticism, and the challenging of ideas were valued.
- Students were involved in the communication of their ideas to others using a variety of means and media.
- The teacher’s questions triggered divergent modes of thinking.
- There was a high proportion of student talk and a significant amount of it occurred between and among students.
- Active participation of students was encouraged and valued.

Explanation Phase
When the whole class is reconvened students are asked to report on their work. Where there has been a confusion of dependent and independent variables, this is addressed with the questions defined in the Exploration Phase. A single, clearly represented white board can be used to pursue the synthesis of the small group work leading into the equations of motion.

The equations of the graph can be worked out verbally first with questions such as, “How far did Suzy go in 7.5 seconds?”, “Can you draw a vertical line on the graph that shows this distance?”, “Can you do the same for Tim?”, “Which graph is steeper?”, “Which student is fastest?, “How do you know?”, “What are the dimensions of speed?” and “What property of the line has these dimensions?”. This can lead to a written model for the motion:

The <u>position</u>-<u>time</u> graph of the motion is a straight line connecting <u>the position at which a runner started</u> to the point at which the race ended and the speed of a runner is seen in the <u>steepness of the line</u>.

The students are then asked to individually assign symbols to parts of this sentence that you have underlined. Then they are asked to write equations using these symbols for each runner with values for symbols that do not change during the race.

The Purpose of the Explanation Phase
There is a correct point-slope representation and the whole-class discussion is used to elicit it from the students' work. This illustrates an RTOP factor that leads to some of the largest performance gains: The focus and direction of the lesson was often determined by ideas originating with students. The alternative to a discussion would be for the teacher to present the correct result with the message that ideas generated by the students had no value since they could not be used to develop a useful result.

The Elaboration Phase
To deepen conceptual understanding students should make use of new ideas in contexts that are slightly more complex. More complex graphs can be developed by students in small groups for motion that is piecewise.

To illustrate, students could be given this story with an engaging "hook": "decide if Terry was in a greater hurry to eat or to see his girlfriend."

> Terry was really hungry so he decided to go to Rick's gas station to buy chicken strips. Starting at his house by the ball fields he jogs the 2200 m in 200 s then arrives at Ricks. He stays 8 minutes at Ricks while he eats his chicken strips. On the way home he decides to stop by Brenda's (his girl friend) house that is positioned between his house and Rick's. It takes him 98 seconds to get to his girl friend's house that is 900 m from Rick's. He visits her for 5 minutes before going home. It takes him 190 s to arrive home.

Students are asked to add annotation to the graph that indicates answers to these questions:

- When Terry stayed in one place what did the graph do?
- When Terry started back toward his home what did the graph do?

The instructor monitors progress by visiting the groups. There is a common misconception that might be spotted: Some students want to assume that as Terry is going back toward his house the graph should reverse and return to the starting point. This is a good time to emphasize that the time is independent and will march on. If the graph went backwards so would time.

When small group work is complete the instructor reconvenes the class and asks for reports from selected groups that lead to a full description of the motion. The representation of the time interval for each of the segments should be expressed first in language, for example as, "for the time between 200 and 680 seconds". Students should be asked to develop the conventional notation through the use of guiding questions, "How do we say mathematically, for a time greater than 200 seconds and less than 680 seconds?"

The Purpose of the Elaboration Phase
The small group work followed by whole-class discussion supports those RTOP factors that have already been identified. In particular, the emphasis on translation between verbal, symbolic and graphical representations, if consistently repeated throughout the

course, will help students to become aware that there are multiple ways to solve problems that lead to the same answer.

Evaluation Phase

Some of the best evaluations of student work have no grade attached. The reward for the student is that they receive feedback. It is useful for this feedback to come from other students. Each student is asked to develop a motion scenario and have a classmate read the scenario and graph it. They then show their graph to a third student to see if they can interpret it and get the same scenario as was originally written.

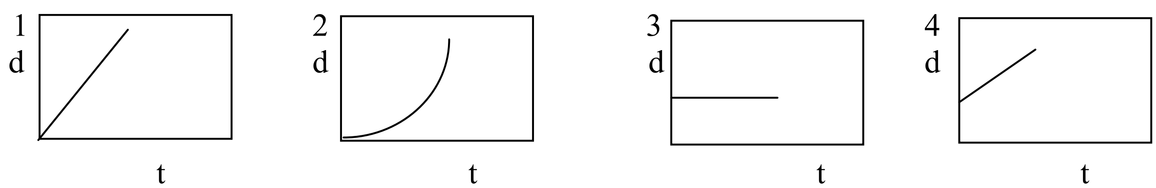
It is important for the students and the teacher to be on the same page about the purpose served by the work that is done. Often it is best for the teacher to identify these learning objectives at the beginning of the work. In some cases, such as in this activity that doesn't really work very well because the tasks are 'uncovering" concepts. Instead of telling the students what the objectives are before starting, the students can be asked to identify the learning objectives in a whole-class discussion after the work is complete. To motivate that discussion it might be useful to tell them, "There is a test tomorrow on what we did today. The test is already written. What is on the test?" Guide the students by reminding them of a segment of the work and asking, "Why did we do that?" When they walk in the next day they are hopefully ready for a test.

This work has prepared the students for a second day in which Physical Science students collect and analyze data using the handout that follows the Motion Assessment. Physics students are given a more challenging task involving the construction of a programmable strategy to monitor the progress of a robot. This provides an opportunity to review content this is prerequisite.

Physical Science
Motion Pre-Lab Assessment

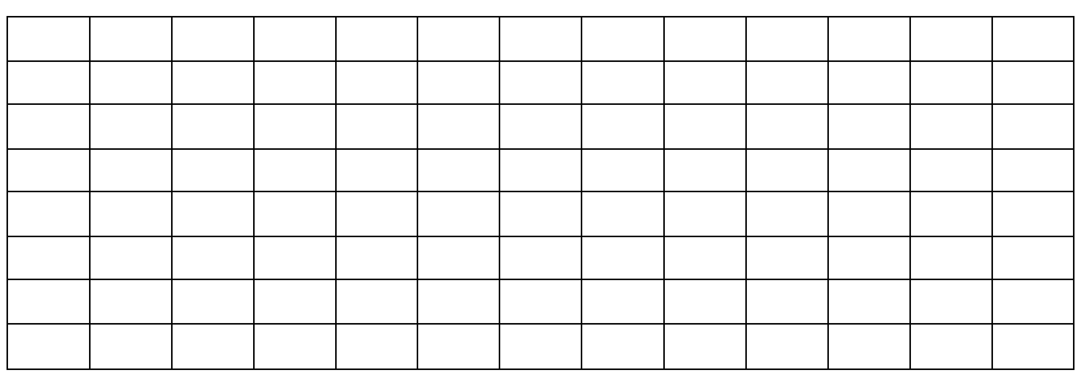
Work together with your team to answer these questions. When you have an answer that you all agree with, put your answer on the white board so that everyone can see it. The teacher will come around and talk to you about your answers.

1. Compare and contrast speed and velocity:
2. How do you find slope of a line?
3. What is the reference point on the graph on the board at the front of the class?
4 What is the independent variable in that graph and why is it called ‘independent’?
5. What is the dependent variable in that graph and on what does it depend?
6. What is the slope-intercept form of that graph and what do the variables represent?
7. In a motion graph if the object can start somewhere beside the reference point, how would this be indicated on a graph? How is it indicated in the slope-intercept formula?
8. Analyze the graphs of distance versus time below and match them with the letter that best describes their motion.



1. The motion is not showing a constant velocity.
2. The motion is at a constant velocity starting from d=0
3. The motion is at a constant velocity of zero starting from d=5 m.
4. The motion is at a constant velocity starting from d=5 m.

9. Draw a graph below to match the story.
From the starting point John walks 15 m in 5 s. Then he stands still for 4s. The he returns to the starting point in 7s.



Constant Velocity
Physical Science

You will collect and analyze data on constant velocity motion. When you have finished this work you should be able to:
• create and interpret graphical representations of data
• determine the independent and dependent variables in a graph and in an experimental procedure
• describe the relationship between the reference point for the measurement of position and the y-intercept on a graph of distance versus time
• describe how slope represents rate of motion

Also, your work will need to demonstrate the ability to:
• persist in the careful completion of a task
• collaborate with members of your team
• clearly communicate your process and your conclusions
• support your conclusions with evidence including the consideration of sources of error

Equipment: Constant velocity vehicle
chalk
Meter stick
Stop Watch

Data Collection:
1. Mark a beginning spot on the pavement or gym floor.
2. One student keeps time while another marks the progress of the vehicle on the ground. At a fixed time interval (we will use 2 or 5 seconds) the timer will yell mark and the marker will mark the spot the vehicle is at. We will do this for 30 seconds.
3. Then the students will measure the distance from the beginning mark and the next mark. That is the distance the vehicle traveled in 5 (or 2) seconds. Measure and record the distance between each mark for each time interval.
4. Create a graph of the information. Using the graph determine the speed of the vehicle.

Begin the work by reading over this outline of a procedure in collaboration with your group. Make a sketch on a white board of the setup with annotation that indicates the number line to be used to define values of position. Provide annotation on the sketch about the collection of data on the time variable and describe how the measurement of time and position will be coordinated.
Also, discuss the roles of each member of the group and what can be done so that the motion is most nearly done with a constant velocity. Include these on your white board.

In your lab notebook construct a table like the following one to collect the data and add rows as needed:

Time (s)	Distance (m) between marks	Total Distance (m)
0s	0m	0m

When everyone in the group has the copied the data into their lab notebooks each member should use the following to construct a graph of the data using correct independent and dependent variables.

When the graphs are complete compare your graphs within the group. Are there differences? Come to an agreement as a team and then sketch the graph, with necessary annotation on the whiteboard. The teacher will visit your group to discuss your results and you may be asked to present to the entire class. So make sure that the graph and annotation can be seen clearly by everyone in the room.

Work out answers to the following questions within the group. Be ready to discuss your answers with the teacher.

- Are there data points that are not as close to the line as others? Can you account for this? How can the data collection be improved?
- Express the data using a best-fit line and equation representing that line. Record both on the white board.
- Suppose that the motion began (t=0) at 1 meter (d =1 at t=0). How would the graph change? Would this change the way that the car moved?
- Suppose that the car moved three times as fast. How would the graph change? Add this line to the graph on the white board.
- Suppose that the slope of the graph was multiplied by –1. Would the motion represented by the new line have the same speed as the old one? Would the velocity of the new time be the same as the old line?

This rubric will be used to evaluate your work:

Description	1 point No work And objective not met	2 points Work done but objective not met	3 points Work done and objective partially met	4 points Work done and objective fully met	5 points Work done and objective is exceeded
Careful data collection and analysis					
Collaboration within group					
Communication of process and results					
Support of conclusions with evidence					
Creation of graphical representation of data					
Identification of variables					
Description of relationship between intercept on graph and motion					
Description of relationship between slope on graph and motion					

Physical Science
Motion Post-Lab Assessment

1. What is your independent variable? ________________________________

   What axis do you graph it on? ________________________________

2. What is your dependent variable? ________________________________

   What axis do you graph it on? ________________________________

3. Was the speed of your vehicle constant? ____________________________

   How can you tell? ____________________ ____________________

4. If the speed was not constant, what factors would have caused this?

   ____

   ____

5. What could be possible causes of experimental error?

   ____

   ____

6. Use the data provided to construct a graph with appropriate labels and add a best-fit line.

7. Use the slope formula to determine the slope of your line and show your work.

8. What is the speed of the vehicle whose motion produced this data? Show your work.

9. Write an equation for the graph in slope intercept form.

10. If the car had started at the 2-meter mark instead of the zero what would the equation of the graph be?

Constant Velocity Homework
Physical Science

Work the following problems and be prepared to discuss them at the start of the next class period.

1. A woman counts her steps as she walks. In 30 s she walks a total distance of 120 m and takes 60 steps.

Steps	Time (s)
4	2
10	5
16	8
60	30

What is the average length of her stride? ________________________

Using the formula v = d/t what is her velocity? ___________________

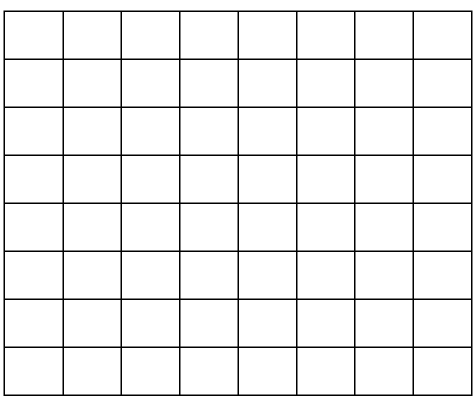
Using the slope formula (show work) what is her velocity?

Then she counted her steps and obtained data in the table at the right.

What is the dependent variable? ____________________________

What is the independent variable? ____________________________

Write an equation for the data in the table in slope intercept form.



2. A car starts at the 5 m mark and travels 15 m in 5 s putting her 20 m from the zero point at 5s. Graph the motion and write the equation in slope-intercept form.

3. A boy runs at 4 m/s and at 5 s he is at the 24 meter mark. Graph the motion and write the equation in the slope-intercept form.

4. Given the equation y = 3x + 4 describe the motion of the object giving velocity and starting position:

5. Read the following story and write an equation (in slope intercept form) that describes

the motion of each person.

Kim and Steve are having a race. Kim knows she is faster so she gives Steve a 3 m head start. Kim runs at 4 m/s and Steve runs at 3 m/s.

Write Kim's equation and explain how you obtained it:

Write Steve's equation and explain how you obtained it:

6. Jamal runs 30 meters in 5 seconds. What is Jamal's velocity? (Show Work)

---

7. Timmy and Tonya race toy cars. Both cars run at 2 m/s. Tonya starts 1 m in front of Timmy. What do we know about the slopes of the graphs of their motions?

Write the equations that describe the motions of these cars.

Tommy's: ___________________________

Tonya's: ___________________________

---

8. When looking at a graph showing the motion of more than one object, you see parallel lines. What does that tell you about the objects?

Constant Velocity
Physics

In order to investigate more complex motion it will be important to be able to use the concepts and skills that were used in your Physical Sciences course. Also, it is very important for success in this course to acquire the habit of working with other students to design experiments and develop strategies. When you have finished this work you should be able to:

- create and interpret graphical representations of data
- design the collection of data and develop a strategy to analyze that data to find a relationship between a pair of independent and dependent variables
- describe the relationship between the reference point for the measurement of position and the y-intercept on a graph of distance versus time
- explain how slope represents rate of motion in terms of defining relationship between velocity, position and time

Also, your work will need to demonstrate the ability to:

- persist in the careful completion of a task
- collaborate with members of your team
- clearly communicate your process and your conclusions
- support your conclusions with evidence including the consideration of sources of error

You are asked to develop a sequence of steps that could be used to program a robot so that it could monitor its speed as it moved in a straight line over a flat surface. This elementary task would then be extended to develop all essential navigational routines.

Work out your answers to the following in your group. When everyone agrees put your answers on the white board so that you can discuss then with your instructor when he or she visits the group.

- What are the variables that must be measured as the robot moves?

- These data will be stored in memory as an array; a table with columns for each of these variables. Construct a data table that represents this array with column headers that identify the variables.

- What are the steps that should be programmed?

Work out answers to the following questions within the group. Be ready to discuss your answers with the teacher.

- As the robot moves some wobbly and jerky motion is expected. How would this affect the quality of the monitored speed value if only two sequential points in time were used to calculate the speed?
- Describe a strategy to improve the quality of the monitored speed value using what you know about the best-fit representation of position-time data.
- Suppose that an external force caused the robot to change direction. What conditional statement could you add to your sequence to allow the robot to calculate velocity?

Ball Toss

Doug Reed
Pulaski High School
Little Rock, Arkansas

The Setting

Pulaski Academy is a private school serving pre-K through twelfth grade. The total enrollment is about 1400 students. Physics classes at Pulaski Academy meet on a block schedule every other day for 80 minutes and have between 12-24 students.

Overview

This activity involves exploring the kinematics of a ball toss into the air. This is a structured inquiry with qualitative data analysis. It gives instruction on how to use *Logger Pro* including how to analyze data and graphs using the program. This activity would apply to Pre-AP and AP Physics B and C. Physics classes at Pulaski Academy meet on a block schedule every other day for 80 minutes and have between 12-24 students.

Materials

Vernier Motion Detector
Volleyball or basketball and light beach type ball
Computer with *Logger Pro* installed
Vernier computer interface – *Logger Pro*
Most of these can be obtained at www.vernier.com

Arkansas Standards

P.6.PS.3 Compare and contrast among speed, velocity and acceleration
P.6.PS.4.c Where a = acceleration, v = speed (velocity), delta t = change in time, delta v = change in velocity, t = time and d = distance
P.6.PS.5. distance versus time (d-t)
P.6.PS.5.b velocity versus time (v-t)
P.6.PS.5.c acceleration versus time (a-t)
MF.1.P.5.a $v(f)^2 = v(i)^2 + 2a\,\Delta y$
NS.10.PS.4 Gather and analyze data using appropriate summary statistics
NS.12.PS.1 Use appropriate equipment and technology as tools for solving problems
NS.12.PS.2 Collect and analyze scientific data using appropriate mathematical calculations, figures, and tables
NS.19.P.2 Manipulate scientific data using appropriate mathematical calculations, charts, tables, and graphs

Learning Objectives

- Be able to interpret graphs of position, velocity, and acceleration data as a ball travels straight up and down
- Be able to construct a table to organize data.
- Be able to analyze the position *vs.* time, velocity *vs.* time, and acceleration *vs.* time graphs.
- Be able to determine the best-fit equations for the position *vs.* time and velocity *vs.* time graphs.
- Be able to determine the mean acceleration from the acceleration *vs.* time graph.
- Be able to predict shapes and location of graphs of position, velocity, and acceleration time graphs on a Cartesian coordinate axis.

Learning Sequence
This lab provides a guided-inquiry approach to exploring the toss of a ball and the behavior of three graphs as the ball is being held, thrown, in free fall (free from human touch) and catching and then holding the ball once again. The three graphs are position, velocity, and acceleration-time graphs.

Initiation Phase
From the RTOP rubric, it is a good idea to begin by showing the students how to throw the ball and encourage them to make predictions based on what they already know. A short discussion of the *Logger Pro* program and how it works would be a good idea also. Both can be done with a demonstration of the ball toss in relation to the equipment followed by the students recording their prediction on a printed copy of the three graphs.

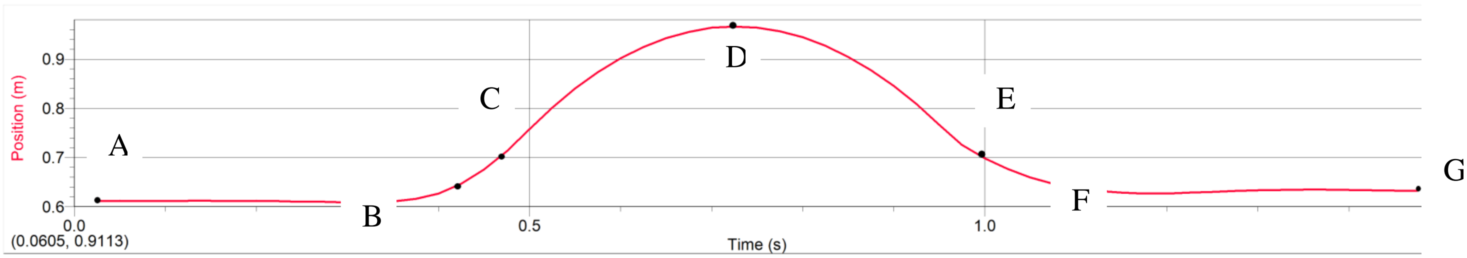
Prediction and Experimentation Phase
The students should make predictions before they start data gathering. By checking that predictions are made and based on reasoning the value of data gathering should be increased. The majority of College Ready students claim to understand the purpose of steps in a laboratory procedure. Questions like "what does this graph mean?" and "What is the ball doing here on this graph?" can be used to focus attention on the details of the motion and the variables being graphed. It is best not to correct student work during the prediction phase. The students will revisit their graphs following the analysis part of the lab. The same questions can be used to assess their understanding of the data being collected. And the simple question, "How were these data obtained?", can reveal understanding of the connection between the results and the data collection technique.

Synthesis and Evaluation Phase
The synthesis phase comes at the end when they pull all of the data together in data tables, look over the data and evaluate their value of *g* ($9.8\ m/s^2$). Students should be asked, during a whole-class discussion, to summarize the differences between their predicted graphs at the beginning and the results of their analysis. They are further asked to present and support conclusions about and evaluate *g* for both the heavy ball and the light ball and the motion of a ball that bounces after being dropped.

Post lab Test – not graded on accuracy but completion

The following three graphs are graphs of a ball being tossed into the air.

1. Given the following graph, answer questions 1 – 3



1. For what sections is the ball being held?

   a. A to B
   b. B to C
   c. C to E
   d. F to G
   e. Both a and d are correct answers

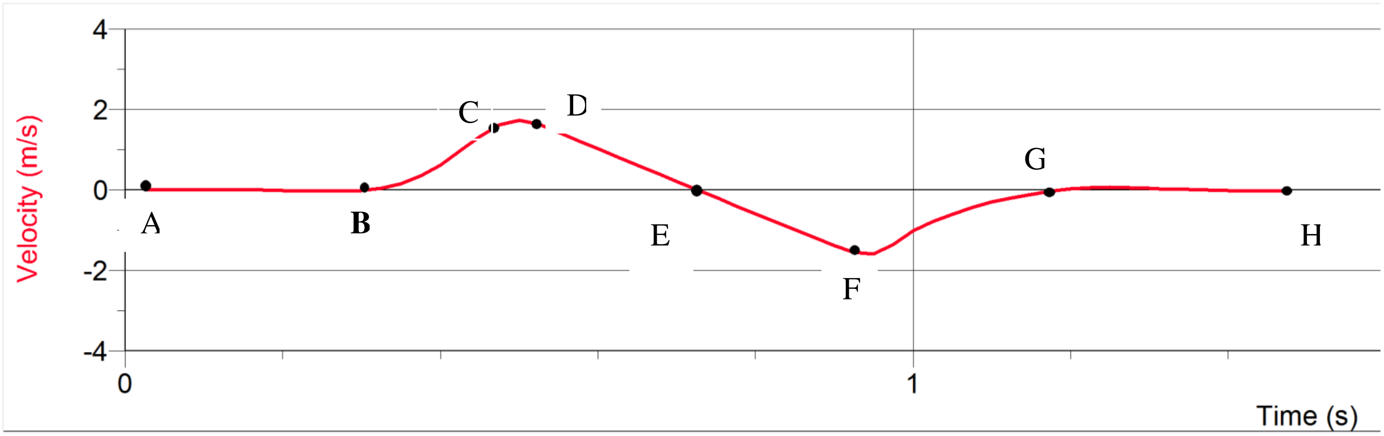
2. When is the ball being thrown?

   a. A to B
   b. B to C
   c. C to E
   d. F to G
   e. Both b and d are correct answers

3. At what point is the ball at the top of its motion?

   a. A
   b. B
   c. C
   d. D
   e. F

Answers question numbers 4 – 7 using the following graph



4. For what section is the ball in free fall without any human touch?

    a. B to C
    b. D to F
    c. B to F
    d. B to G
    e. C to G

5. For what section was the ball being caught?

    a. A to B
    b. B to C
    c. E to F
    d. F to G
    e. F to H

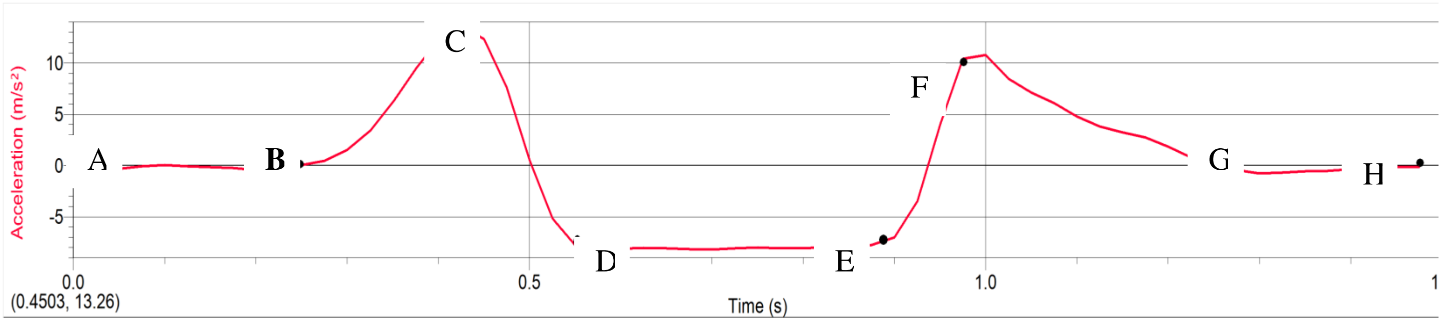
6. At what point is the ball not moving?

    a. C
    b. D
    c. E
    d. F

7. What is the ball doing for section E to F?

    a. Not moving
    b. Speeding up
    c. Slowing down
    d. Being caught
    e. Being held

Use the following graph to answer questions #8 – 10



8. For what section is the ball in free fall without any human touch?

   a. A to B
   b. C to D
   c. C to F
   d. D to E
   e. B to G

9. For what section was the ball being thrown?

   a. A to B
   b. B to D
   c. E to F
   d. F to G
   e. F to H

10. At what point is the ball being held?

    a. A to B
    b. B to C
    c. D to E
    d. E to F
    e. B to D

Rubric for Ball Toss lab

Indicator	***Points possible***	***Points earned***
Did they predict?	5	
Did they print out and label the position time graph	2	
Did they print out and label the velocity time graph	2	
Did they print out and label the acceleration time graph	2	
Curve fit the position time graph	3	
Curve fit the velocity time graph	3	
Curve fit the acceleration time graph	3	
Heavy ball data with average (at least 5 trials)	5	
Light ball data with average (at least 5 trials)	5	
Data table for graph #s	5	
Data table for heavy ball #s	5	
Data table for light ball #s	5	
Conclusion with percent error and explanation of why	5	
TOTAL	50 points	

Ball Toss

In this experiment, you will use a Motion Detector to collect position, velocity, and acceleration data for a ball thrown straight upward and learn to analyze the data.

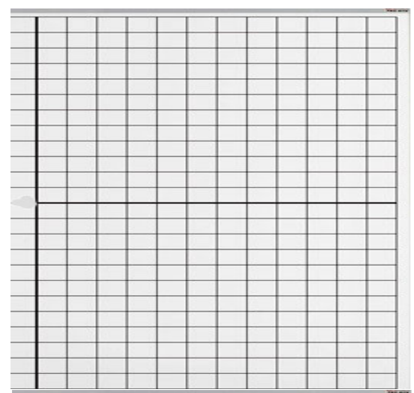
objectives

- ✓ Be able to interpret graphs of position, velocity, and acceleration data as a ball travels straight up and down
- ✓ Be able to construct a table to organize data.
- ✓ Analyze the position *vs.* time, velocity *vs.* time, and acceleration *vs.* time graphs.
- ✓ Determine the best fit equations for the position *vs.* time and velocity *vs.* time graphs.
- ✓ Determine the mean acceleration from the acceleration *vs.* time graph.

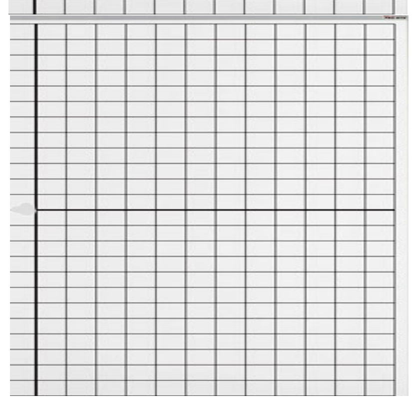
Materials

computer	Vernier Motion Detector
Vernier computer interface – Logger Pro	volleyball or basketball

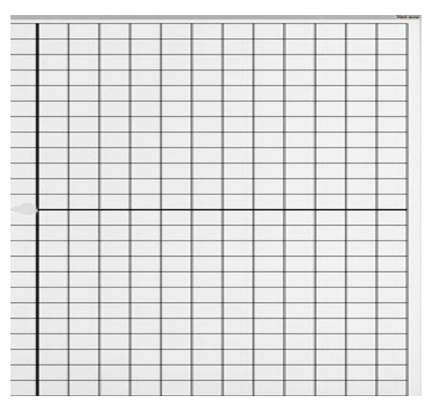
Predictions – please make a prediction BEFORE you do the lab! Print this first page so you can write on it.



1. Think about the changes in motion a ball will undergo as it travels straight up and down. Make a sketch of your prediction for the position *vs.* time graph. Describe what this graph means.



2. Make a sketch of your prediction for the velocity *vs.* time graph. Describe what this graph means.



3. Make a sketch of your prediction for the acceleration *vs.* time graph. Describe what this graph means.

Procedure

1. Connect the Vernier Motion Detector to the DIG/SONIC 1 channel of the interface.

2. Place the Motion Detector on the floor.

3. Open the file "06 Ball Toss" from the *Physics with Vernier* folder.

4. In this step, you will toss the ball straight upward above the Motion Detector and let it fall back toward the Motion Detector. This step may require some practice. Do not let the ball hit the sensor. Hold the ball directly above and about 0.5 m from the Motion Detector. Click **Collect** to begin data collection. You will notice a clicking sound from the Motion Detector. Wait one second, and then toss the ball straight upward. Be sure to move your hands out of the way after you release it.

5. Examine the position *vs.* time graph. Repeat Step 4 if your position *vs.* time graph does not show an area of smoothly changing position. Check with your teacher if you are not sure whether you need to repeat the data collection.

Analysis

You will do this as a group working together. Please discuss and debate with your group members. Only ask the teacher after you have thoroughly debated the questions in your group!

1. Print the three motion graphs separately. The graphs you have recorded are fairly complex and it is important to identify different regions of each graph. Record your answers directly on the printed graphs.

   a) Identify the region when the ball was being held and tossed but still in your hands:

- Examine the position *vs.* time graph and identify this region. Label this on the graph.
- Examine the velocity *vs.* time graph and identify this region. Label this on the graph.
- Examine the acceleration *vs.* time graph and identify the same region. Label this on the graph.

   b) Identify the region where the ball is in free fall (you were not in contact with the ball):

- Label the region on each graph where the ball was in free fall and moving upward.
- Label the region on each graph where the ball was in free fall and moving downward.

   c) Determine the position, velocity, and acceleration at specific points.

   - On the velocity *vs.* time graph, decide where the ball had its maximum velocity, just as the ball was released. Mark the spot and record the value on the graph. Use the examine button to get a value
   - On the position *vs.* time graph, locate the maximum height of the ball during free fall. Mark the spot and record the value on the graph.
   - What was the velocity of the ball at the top of its motion? Record this number on the graph.
   - What was the acceleration of the ball at the top of its motion? Record this number on the graph.

2. The motion of an object in free fall is modeled by $\Delta y = v_i t + \frac{1}{2}gt^2$, where Δy is the vertical position, v_i is the initial velocity, t is time, and g is the acceleration due to gravity (9.8 m/s^2). Look at the position- time graph.

   For the free fall section only, Please record what kind of graph (shape) it is and record what type of graph this is on the graph itself. To fit an equation to your data, click and drag the mouse across the portion of the position *vs.* time graph that is curved, highlighting the free-fall portion only. Click the Curve Fit button, , select the fit from the list of models you think will work with your graph and click Try Fit. Examine the fit of the curve to your data. If it is not a good fit, ask your teacher. Click OK to return to the main graph if the fit is a good one.

   In the model there is only one parameter, *g*. By comparing the value of *g* that was obtained in the curve fit with the known value of *g* the model is experimentally tested.

✓ When you record values (numbers), you need to make a data table to hold the numbers. Read a) through d) below. Then make a data table on green engineering paper for this purpose. Use the Examine button, , if needed

   a) Record the coefficient of the t^2 term (the A number) in the curve fit so that it can be compared to $\frac{1}{2}g$.

   b) Look at the graph of velocity *vs.* time. For the free fall section only, Please record what kind of graph (shape) it is and record what type of graph. Repeat the instructions above in #2 to analyze the graph and get the best-fit equation. What type of equation would you choose? After getting the equation, Record the value the coefficient of the *t* term in your table (the *m* number).

   c) Look at the graph of acceleration *vs.* time. Click and drag the mouse across the **free-fall section** of the motion and click the Statistics button, . Record this value of the mean acceleration value in your table.

   d) Determine the consistency of your acceleration values and compare your measurement of *g* to the accepted value of g. Do this by repeating the ball toss experiment five more times. Average your six slopes to find a final value for your measurement of *g*. Record these numbers in your table. Does the variation in your six measurements explain any discrepancy between your average value and the accepted value of *g*?

3. The ball used in this lab is large enough and light enough that a buoyant force and air resistance may affect the acceleration. To probe this effect you can perform the same curve fitting and statistical analysis techniques, but this time analyze each half of the motion

a) Make a prediction in the form of an inequality about the acceleration of the light, large ball from best-fits to these motions and the acceleration due to gravity.

✓ b) Make a data table on green engineering paper to organize data that can be used to draw conclusions about the effects of air resistance using a light ball or a very light, large ball. Perform the necessary experiments and record your values this table.

c) After a suitable number of trials compare the acceleration to the g you got for the heavier ball. Perform a percent error using g as the accepted value.

4. For a conclusion, go back and compare your predictions with what actually happened. Because of the lab, are you able to think more clearly about the motion of a ball in the air? Perform a percent error for both the heavy and light balls using your values from number 2 and 3 for your experimental numbers and g for your accepted number. List some reasons why your values for the ball's acceleration may be different from the accepted value for *g*. Talk about the heavy ball and the light ball's acceleration a

5. Instead of throwing a ball upward, drop a ball and have it bounce on the ground. Predict what the three graphs will look like, and then analyze the resulting graphs using the same techniques as this lab.

Roller Coaster Physics

Johnny Tarrant
Northside High School
Fort Smith, AR

Introduction

This physics activity centers around high school students designing a world-class roller coaster ride for a super amusement park to be built in the U.S. With modifications, this project can be used for AP Physics, Pre-AP Physics, and Conceptual Physics classes, as well as for physical science students. Because of the design element involved, the activity will probably require 4-5 class periods for a class lasting 50 minutes. The students will gain an appreciation for the many aspects that go into the designing of any amusement ride, especially in the area of human safety.

Materials and Preparation

This activity will require the use of a computer and printer with an Internet connection for the research on large, world-class roller coasters. In addition, the students will need:

- Scientific-level calculators or if available, graphing calculators (TI-84 or equivalent).
- Graph paper
- If available, a KNEX® model roller coaster set (Speed Demon Coaster or equivalent)

Relevant Arkansas Science Standards

The following Arkansas Science Standards are relevant to this activity:

Arkansas Physical Science Standards

P.6.PS.2 Explain how *motion* is relative to a *reference point*

P.6.PS.13 Design an experiment to show conversion of mechanical (potential and kinetic) *energy*

P.6.PS.14 Solve problems by using formulas for *gravitational potential* and *kinetic energy:* $KE = \frac{1}{2}mv^2$ and $PE = mgh$ where KE = kinetic energy, PE = potential energy, m = mass, v = velocity

NS.9.PS.4 Summarize the guidelines of science:

- explanations are based on observations, evidence, and testing
- *hypotheses* must be testable
- understandings and/or conclusions may change with additional empirical data
- scientific knowledge must have peer review and verification before acceptance

NS.9.PS.2 Compare and contrast *hypotheses*, *theories*, and *laws*

NS.10.PS.1 Develop and explain the appropriate procedure, *controls*, and *variables* (dependent and independent) in scientific experimentation

NS.10.PS.3 Identify sources of *bias* that could affect experimental outcome

NS.10.PS.4 Gather and analyze data using appropriate summary statistics

NS.10.PS.5 Formulate valid conclusions without *bias*

NS.10.PS.6 Communicate experimental results using appropriate reports, figures, and tables

NS.12.PS.1 Use appropriate equipment and technology as tools for solving problems

NS.12.PS.2 Collect and analyze scientific data using appropriate mathematical calculations, figures, and tables

Arkansas Physics Standards

MF.2.P.2 Resolve two-dimensional *vectors* into their *components:* $d_x = d\cos\theta$ and $d_y = d\sin\theta$

MF.2.P.3 Calculate the *magnitude* and direction of a *vector* from its *components* $d^2 = x^2 + y^2$ and $tan^{-1}\theta = x / y$

MF.4.P.1 Calculate net work done by a constant net force: $W_{net} = F_{net} d\cos\theta$ where $W_{net} = work$

MF.4.P.2 Solve problems relating kinetic energy and potential energy to the *work-energy theorem:* $W_{net} = \Delta KE$

MF.4.P.3 Solve problems through the application of conservation of mechanical energy: $ME_i = ME_f$ and $\frac{1}{2}mv_i^2 + mgh_i = \frac{1}{2}mv_f^2 + mgh_f$

MF.4.P.4 Relate the concepts of time and *energy* to power

MF.4.P.5 Prove the relationship of time, *energy* and power through problem solving: $P = \frac{W}{\Delta t}$ and $P = Fv$ where P = power; W = work; F = force; v = velocity; and t = time

NS.16.P.2 Compare and contrast the criteria for the formation of hypotheses, theories and laws

NS.16.P.3 Summarize the guidelines of science:

- results are based on observations, evidence, and testing
- hypotheses must be testable
- understandings and/or conclusions may change as new data are generated
- empirical knowledge must have peer review and verification before acceptance

NS.17.P.4 Gather and analyze data using appropriate summary statistics (e.g., percent yield, percent error)

NS.17.P.3 Identify sources of bias that could affect experimental outcome

NS.17.P.5 Formulate valid conclusions without bias

NS.18.P.1 Recognize that theories are scientific explanations that require empirical data, verification and peer review

NS.19.P.1 Use appropriate equipment and technology as tools for solving problems

NS.19.P.2 Manipulate scientific data using appropriate mathematical calculations, charts, tables, and graphs

Student Learning Objectives

A student in a Physical Sciences course will be able to:

- Explain how *motion* is relative to a *reference point*
- Gather and analyze data using appropriate summary statistics
- Communicate experimental results using appropriate reports, figures, and tables
- Use appropriate equipment and technology as tools for solving problems
- Collect and analyze scientific data using appropriate mathematical calculations, figures, and tables

In addition, a student in a Physics course will be able to:

- Resolve two-dimensional *vectors* into their *components*

- Calculate the *magnitude* and direction of a *vector* from its *components*
- Calculate net work done by a constant net force
- Solve problems relating kinetic energy and potential energy to the *work-energy theorem*
- Solve problems through the application of conservation of mechanical energy
- Relate the concepts of time and *energy* to power
- Compare and contrast the criteria for the formation of hypotheses, theories and laws
- Summarize the guidelines of science: results are based on observations, evidence, and testing and hypotheses must be testable
- Identify sources of bias that could affect experimental outcome
- Formulate valid conclusions without bias
- Recognize that theories are scientific explanations that require empirical data, verification and peer review

The Learning Cycle

This project asks students to apply the principle of energy conservation to understand the operation of high-speed roller coasters and to play a role as a member of design team to develop a proposal for an even more exciting roller coaster. This project utilizes the *5 E's learning cycle:* engagement, exploration, explanation, elaboration/extension, and evaluation. While each of these phases is present in the project, the activity also serves as an elaboration of a larger learning sequence. Students come to this activity as a capstone for the study of kinetic and potential energy, heat, and work.

To initiate the project activity, the students, as a class, will be given the following challenge:

> "The company where you work is making a bid on a 30 million dollar project to build a roller coaster with a velocity greater than the Formula Rossa roller coaster at Ferrari World Abu Dhabi. Your role as a member of the physics team is to develop and test a design equation for the roller coaster. This equation will be used by the car design team and by the hydraulic launch team. They need to use the equation to make decisions about the mass of car materials, power requirements, track design, and maximum speed beyond the first peak."

The students will form small work groups and will research current roller coaster designs, such as the Formula Rossa on the internet. Each group will come up with a preliminary model on paper including possible design equations. Finally, the students discuss their results as a class, with each group reporting on their results.

Engagement Phase

The students should be introduced to the project by showing the following video clip where the roller coaster designers did not optimize all their design factors: http://www.youtube.com/watch?v=oNYzj6tw7XA&feature=related . This video clip should start a conversation where the instructor can introduce the design challenge as mentioned earlier.

Following the discussion of the challenge, questions should guide the whole-class discussion toward the following variables:

- Force along the ramp
- Mass of the cart
- Location of a reference point for the measurement of position
- Distance along the ramp from the reference point to the highest point
- Height at the top of the ramp relative to the reference
- Velocity of the cart at the bottom of the ramp

A drawing of the system should be developed collaboratively with the student teams. It should break the problem up into the two events: cart ascends and cart descends. Details of the measurements needed should be discussed.

RTOP and the Engagement Phase

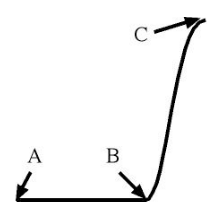
In traditional teacher-centered classroom, the instructor presents the ideas or concepts to be learned, while the student takes on a passive role of note-taker or listener; in a lab class, the student then tests the concept by performing an experiment where the outcome is pre-determined. However, as one can note from the RTOP factors discussed during the *College Ready* workshops, in a reformed teaching type classroom, the highest value is given to students learning a concept through observation of a phenomenon and discussing what they saw and its importance. That is the approach taken here.

Exploration

As a starting point at a physical science level students could be encouraged to explore a website, http://www.learner.org/interactives/parkphysics/coaster/, that steps them through variations in the variables that should be in their models. Combinations of parameters are rated in terms of their safety and thrill without developing the model that is the goal of this more advanced project.

Models describing relationships between variables during each of the two events should be brainstormed by the student teams. Remember, since one of the learning objectives is the application of the idea of work done by a force, then you may need to redirect their thinking through your questions and reference to the problem-based scenario. Remind them that the car design team needs a design equation that contains mass.

A good way to initiate the brainstorming and to restrict the model-building to just the critical first segment of the track, is by asking them to restrict their initial model to the points on this drawing.



Possible models that refer to the points in the sketch are:

$$mgh_C + mv_C^2/2 - mgh_A - mv_A^2/2 = Q_{AC} + F_{ext,ascent}L_{BC} + F_{ext,launch}L_{AB}$$

$$mgh_C + mv_C^2/2 - mgh_B - mv_B^2/2 = Q_{BC} + F_{ext,ascent}L_{BC}$$

$$mv_B^2/2 - mv_A^2/2 = Q_{AB} + F_{ext,launch}L_{AB}$$

In the case of student difficulty with developing mathematical models, these equations could be written on a white board a piece at a time, during discussion with a small group. This would allow the students to relate the model to equations they have dealt with in prior lessons. In these models the values of Q (thermal energy transfer, heat) will be negative.

It is important for the students to consider human safety factors in their testing, just as engineers and technicians would do in the real world. The instructor should check for this during visits with the lab groups. If safety factors are not involved in a group's testing and analysis, then the instructor should ask, "Why don't I see any human safety considerations in your testing?" Some good websites that help students understand how safety is involved in rollercoaster design are:

- AP physics students might look at a more sophisticated discussion at : http://www.exponent.com/Making-Thrills-Safer--The-Evolution-of-Todays-Roller-Coasters/
- For physical science, and basic physics classes, http://www.darylscience.com/Demos/RollerCoaster.html

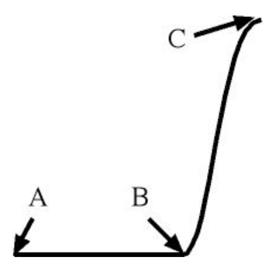
The exploration phase supports following RTOP factors:

- Students made predictions, estimations and/or hypotheses and devised means for testing them.
- This lesson encouraged students to seek and value alternative modes of investigation or of problem solving.
- The focus and direction of the lesson was often determined by ideas originating with students.

Explanation

When completed, the groups can apply their model using from the Kingda Ka roller coaster at Six Flags, NJ (http://www.sixflags.com/greatadventure/rides/kingdaka.aspx):

Height of first peak: 456 feet
Height of first trough: 15 feet
Top Speed: 128 mph
Time to reach top speed: 3.5 seconds
Cars: 5 cars each with two passengers
Maximum power for launch: 20,800 horsepower



Have each group consider the points shown in the diagram to the right: A is where the passengers get on board, B is at the end of the horizontal launch track, and C is at the top of first peak where the cars may momentarily pause before descent. A hydraulic launch design in which a series of motors accelerate the cars along a horizontal track in front of the first peak is used.

The questions listed below can be either posted on a board in the front of the classroom or given as a handout to each group to guide their analysis (In addition, the questions can be used as part of a whole-group main discussion or inter-group discussions. Instructors can adapt these questions to the level of class they are working with. Each group is expected to post their calculations as supporting evidence on a whiteboard.

Questions should be based on the following variables:

- What is the maximum force along the horizontal launch track?
- What is maximum mass of each of the cars?
- What is the velocity of the cart at the first peak with these values of force and mass?
- Are your design equations consistent with the design of the *Kingda Ka*?

If this project is used in an AP Physics class, the instructor may wish to use a higher-level set of questions to guide student work:

- What interactions between the system and its surroundings can produce heat?
- What could you observe that would provide evidence of these interactions?
- What fraction of the energy at B is transferred out of the car system as heat between points B and C?
- What mass of cars and passengers can be supported if there is no heat transfer between points A and B?
- If a typical launch requires only one half of the maximum power, what is the mass of cars and passengers for a typical launch, assuming no heat transfer between A and B?
- If a typical launch requires only one half of the maximum power, what is the mass of cars and passengers for a typical launch, assuming no heat transfer between A and B?
- How much work is done if half of the maximum power is used for the 3.5s launch?
- Suppose that gasoline is used to perform this work. The energy density of gasoline is about 34 MJ/L. How many liters are needed to launch the cars?
- The transfer of this amount of energy in a very short time involves a lot of power. How long would it take to evaporate one kg of water with the maximum power? The heat of vaporization of water is 2260 kJ/kg.
- Voltage fluctuations are a concern for the stability of the electric grid that provides power to the roller coaster. In 2007 passengers hung upside down for a half-hour on a roller coaster in Hot Springs, Arkansas due to a power outage. What fraction of the output of a 500 MW nuclear power plant is required to launch the *Kingda Ka* at maximum power?
- The amusement park has to pay for this power. In 2010 the cost of commercial power in New Jersey is 14 cents per kilowatt-hour. How much does it cost for a typical launch of the *Kingda Ka*?

RTOP and the Explanation Phase
Having the students utilize the above information during the explanation phase should support the following RTOP learning factors:

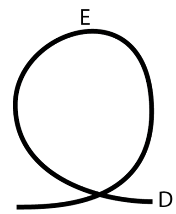
- There was a high proportion of student talk and a significant amount of it occurred between and among students.
- Students were actively engaged in thought-provoking activity that often involved the critical assessment of procedures.
- Students were encouraged to generate conjectures, alternative solution strategies, and ways of interpreting evidence.

The level of student thinking is often guided by the level of questions brought out by the instructor. For AP level students, the questions above can lead to a deeper understanding of the physics involved in roller coaster design. In addition, the RTOP factors below are supported:

- Students were actively engaged in a thought-provoking activity that often involved the critical assessment of procedures.

Elaboration Phase

Missing or misinterpreted parts of their design of the first segment of the roller coaster should have been uncovered in the Explanation Phase. Students should now go back to the drawing board, revise their initial model, and extend the model to include a descent segment.



The descent segment of a roller coaster typically has a loop. By encouraging them to incorporate this feature (or by asking them to return to the interactive roller coaster design website cited above), they will be led to add the application of force in a circular motion to their models. The critical point requires that the contact force of the track on the car not vanish. This introduces relationships between the car velocity and the height at the top of the loop and the condition of the non-zero contact force.

$$mgh_E + mv_E^2/2 - mgh_D - mv_D^2/2 = Q_{DE}$$

$$2mv_E^2/(h_E - h_D) > mg$$

If groups are not recalling the free-body diagram that they studied in problems involving loops such as this, the roller coaster design website might be more effective than attempting to re-teach the concept. Combinations involving loops that are too high receive an "unsafe" rating. The class could be pulled together and this outcome displayed. The students can then be asked to compare an unsafe loop with a safe loop as a starting point for the needed connections.

RTOP and the Elaboration Phase

There is no "correct answer" for the roller coaster design. There are only answers that lead to greatest heights and highest speeds that are consistent with the assumptions that are made. In discussions with small groups the instructor can emphasize the student's role:

- The teacher acted as a resource person, working to support and enhance student investigations.

Evaluation Phase

If this project is done by a physical science class, the evaluation will be a pertinent question on their unit test. The question will involve calculating the acceleration of a roller coaster car of given mass at the bottom of the ramp leading to the first hill. For physics students, the question would be extended to calculating the acceleration of the car going up the hill; the students will be expected to evaluate the acceleration and what the acceleration should be at the peak of the hill. AP level students could be challenged to find kinetic and potential energy levels as well as values of work done by the car. More advanced classes may be asked to explain the trade-offs in the roller coaster design.

If the instructor uses bell ringer questions at the first of the class period to gain student involvement, a question based on the force of the car or acceleration changes due to the height of the hill could be used. For a more interactive evaluation, a marble run could be set up by student groups, with points given for marble speed and number of turns made successfully.

Alternative Evaluation
This project provides an opportunity for small groups to present their results more formally using a poster or web site.

Student Materials
The following handout can be given to the students in order to initiate the project. Additional information can be provided as the project progresses along the lines of the questions and models outlined in these teacher notes.

Roller Coaster Physics
Student Handout

During our discussion of roller coasters and the physics they follow, we have identified several variables that can be used to predict the roller coaster car's velocity and acceleration, the forces on the human beings riding inside and materials that could handle these stresses. You now have a challenge before you, it reads as follows:

> "The company where you work is making a bid on a 30 million dollar project to build a roller coaster with a velocity greater than the Formula Rossa roller coaster at Ferrari World Abu Dhabi. Your role as a member of the physics team is to develop and test a design equation for the roller coaster. This equation will be used by the car design team and by the hydraulic launch team. They need to use the equation to make decisions about the mass of car materials, power requirements, track design, and maximum speed beyond the first peak."

You will be assigned work groups of four persons each. Your group will be responsible for a model of the roller coaster launch ramp and first hill.

As a team, you will report your model and the report must provide the following information:

1. A sketch of your model.
2. A step-by-step explanation of the procedure you used to design your model.
3. Answers to the following questions about your model:
   - What are the major independent and dependent variables involved?
   - What are the units of these variables?
   - What are the major human safety factors to be considered in this project?

As a team, you will develop and report a strategy to test your model and analyze the data produced by your testing. Consider the following questions as you develop your model:

1. What are the actual forces acting on the roller coaster car and its occupants?
2. Where is your reference point for determining motion?
3. What is the mass of the car loaded and unloaded?
4. What velocity will be needed to launch the car over the hill safely?

After you have designed your model and know the launch variables, you will need to report your findings to the class. Choose a member of your group to present the information.