

High School Mathematics Trajectories: Connecting Opportunities to Learn with Student Performance

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Abstract

This study is part of a larger multi-year comprehensive (K-12) mathematics and science curriculum reform initiative focusing on the connection between implemented and attained high school mathematics curriculum. Students' opportunities to learn mathematics content in two geographically diverse school districts were studied to determine if these are linked with student performance in mathematics. All high school students and their mathematics teachers in both districts provided data for this study. Preliminary findings support the contention that curriculum differentiation exists at the high school level. The data suggest that different content trajectories offer very different opportunities to learn within and between school districts and these content trajectories are linked to levels of student performance in mathematics. The findings have implications for student learning outcomes and curriculum policy.

High School Mathematics Trajectories: Connecting Opportunities to Learn with Student Performance

This study is part of a larger multi-year comprehensive (K-12) mathematics and science curriculum reform initiative focusing on the connection between implemented and attained high school mathematics curriculum. Students' opportunity to learn mathematics content (implemented curriculum) is determined by their course taking patterns developed from enrollment data and teachers' reports of content coverage. Student performance (attained curriculum) is assessed using the assessments from the PROM/SE (Promoting Rigorous Outcomes in Mathematics/Science) project. PROM/SE assessments incorporate the framework and instruments of the Third International Mathematics and Science Study (TIMSS and TIMSS-R).

The high school curriculum, especially in mathematics and science, is the focus of much concern in national education policy debates. Mathematics achievement of high school graduates has captured the attention of diverse constituencies such as employers, institutions of higher education, politicians, and the public at large. (Mass Insight Education and Research Institute, 2004; National Academy of Sciences, 2002). At recent education policy meetings, Schmidt and Ferrini-Mundy (2006) have emphasized the need for curriculum coherence (structure), focus (opportunities to learn) and rigor (cognitive complexity).

At the national level there is growing concern about the math literacy of students who will be graduating from high schools. Numerous studies have concluded that U.S. students do poorly in mathematics and science when compared to students in other countries. It is also apparent that the level of achievement of U.S. students in

mathematics and science is lacking, and that the gap between the achievement of black and white students in schools attended by affluent as well as poor students is widening (National Center for Education Statistics, 1996, 1997, 1998, 2000, 2001).

International studies such as Trends in Math and Science Study (TIMSS) and Programme for International Student Assessment (PISA), (OECD,2004) have demonstrated that the U.S. mathematics and science school curriculums are weak compared to international standards. In terms of math literacy, the results from PISA (OECD,2004) indicate that U.S. high school students were ranked in the bottom quartile of the distribution as compared to other developed nations (24th out of 29 nations). On the 2003 TIMSS assessment, 7 % of fourth and eighth graders in the U.S. attained the advanced level compared to 38% of fourth graders and 45% of eighth graders from Singapore which led the achievement rankings (Gonzales, Guzman, Partelow, Pahlke, Jocelyn, Kastberg & Williams, 2004). Studies also indicate that while students in other countries study algebra and geometry, chemistry and physics, most U.S. middle school students study arithmetic and the descriptive aspects of geology and biology (Schmidt & McKnight, 1995; Schmidt, McKnight & Raizen, 1997; Schmidt, W. H., McKnight, C. C., Valverde, G. A., Houang, R. T., & Wiley, D. E., 1997). To address these problems, there have been efforts to produce high quality national standards (National Council of Teachers of Mathematics [NCTM], 2000; National Research Council [NRC], 1996). Findings of the studies conducted in the international arena have also raised concerns about the educational achievement of high school graduates in the US (Schmidt, 2002).

In Initiatives for High School, Higher Education and Job Training (2006), President Bush articulated a new national goal, “to ensure that every high school

student graduates and is ready for the workplace or college.”(p.1). The proposed initiatives ensure that students access a rigorous curriculum in mathematics and science, are taught by trained teachers, and that high schools are held accountable for students’ performance.

To address the concern regarding the preparation of high school students, several states have made policy changes by increasing Carnegie units required for graduation and specifying higher-level courses that must be completed (Dounay, 2006). Labaree (2000) believes that “measuring educational attainment through seat time and credentials rather than through academic performance” (p.30) erroneously connects class time as an indicator of “curriculum mastery” (p.30). Schmidt (2002) also found that number of units earned may not be a good proxy for the attained curriculum.

Rutherford (2005, as cited in Dounay, (2006)) reported that despite having Geometry and Algebra II, 60% of low-income students, 65% of African American and 57% of Hispanic students in Texas failed the state’s test which covered Algebra I. As she aptly states:

While truth-in-labeling practices in the food industry ensure that orange drink cannot be labeled orange juice without legal ramifications, schools have no such safeguards in place. Algebra I can be placed on any child’s transcript without any guarantee about the content taught or learned. (p1.)

Schoenfeld (2002) maintained that discussions about American students’ mathematical needs must be coupled with consideration of factors that ensure all students have access and opportunity to learn the content. Singham (2003) underscored the need to distinguish between the “political problem of inequality” and “educational problems of student underachievement.”

In the PROM/SE project, the approach to improving the achievement of all children is rather straightforward. As a starting point it is important to understand what students know. Then it can be determined what is expected of students based on the standards and what teachers are teaching. Finally, an effort must be made to improve and align all three of these components.

Another crucial factor in instruction is the role of the teacher. In 1996, the National Commission on Teaching and America's Future (NCTAF) argued that pupil learning can be impacted by what teachers know and do in the classroom, and that teacher preparation may well be a viable tool to improve the quality of education in the U.S. (NCTAF, 1996).

Though this relationship would seem to be self-evident, research has failed to demonstrate a clear link between teacher knowledge and student learning (see Wilson, Floden, & Ferrini-Mundy, 2001). Results of studies such as one conducted by Monk (1994) that do hint at a relationship between what teachers know and what students' learn, provide few specific ideas about how teachers' content preparation might be reformed. Some research also indicates that few professional development programs are content driven (Kennedy, 1998).

The conceptual model used in the present study is adapted from Schmidt et al. (2001, p. 15). When this model was used to analyze data from a large scale international study (involving more than 30 countries) there was evidence of a strong empirical relationship between instantiations of the curriculum and student learning and achievement. *Student learning* in this model refers to the knowledge students have acquired during some specified time frame, such as gains across one grade level.

Teacher knowledge includes knowledge of the subject matter, pedagogical content knowledge, and knowledge of students.

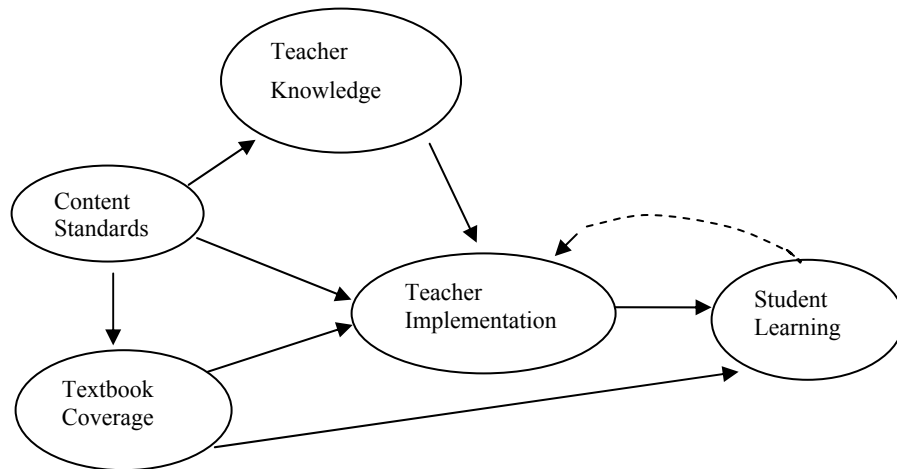


Figure 1. A Model of Student Learning

Each path in the model depicted in Figure 1 represents a complex relationship between the associated nodes. For example, the link between “content standards” and “textbook coverage” suggests that officially articulated content standards affect textbook content. Similarly, content standards may influence teacher preparation at both the pre-service and in-service levels, and thereby influence teaching practice. Classroom time is related to textbook coverage and how teachers choose to utilize this time. The model shows that what gets taught is related to what students learn, and what students learn in turn influences teaching practice.

The model identifies two major and interconnected factors for the improvement of student learning and performance: the curriculum factor and the teacher factor. The curriculum factor includes textbooks and standards, and the teacher factor includes what teachers know about their discipline (content) and their students as learners. In the present study our emphasis is on the curriculum factor which focuses on the relationships among standards, textbooks, teaching practice and their connection to student learning.

The Tri-Partite Model of the Curriculum

Since the First International Mathematics Study in the 1960s, (Travers & Westbury, 1989) the tri-partite model of curriculum (Figure 2) has been used in numerous studies sponsored by the International Association for the Evaluation of Educational Achievement (IEA). This model provides the conceptual basis for the instruments used in the present study. The IEA tri-partite curriculum model defines three different instantiations of the curriculum. First, the *Intended* curriculum is what a system intends students to study and learn. The *Implemented* curriculum is what is taught in classrooms, and finally, the *Attained* curriculum is what students are able to demonstrate that they know.

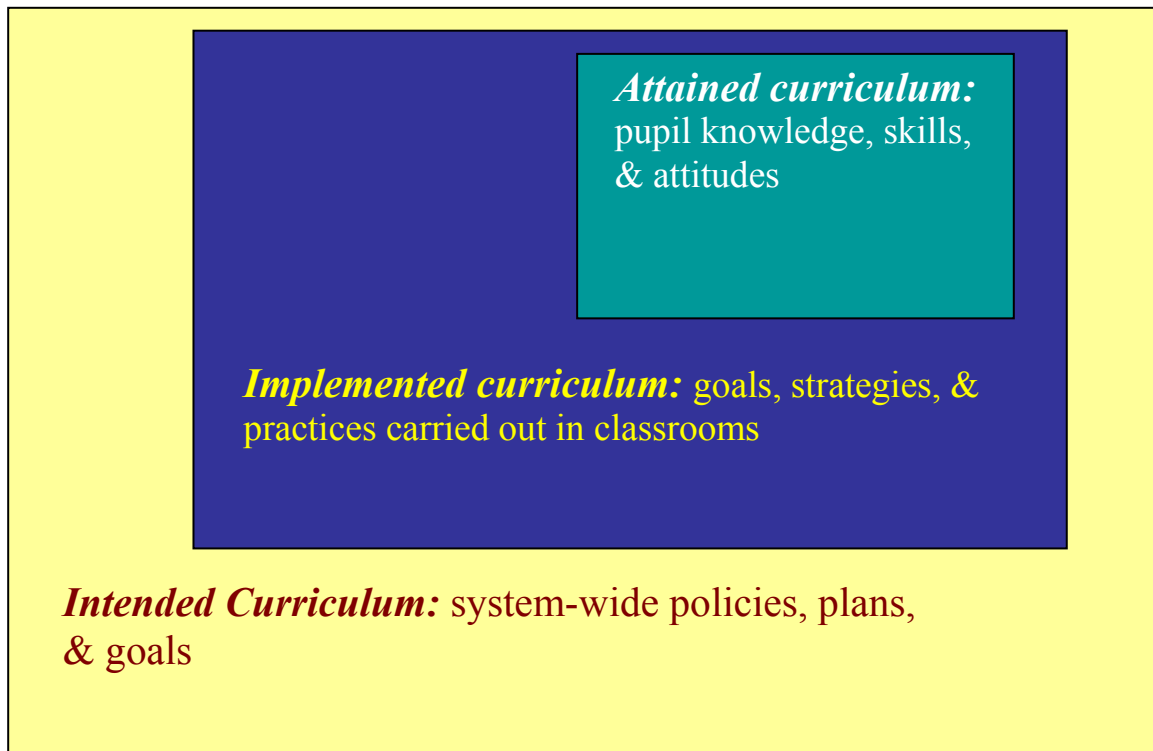


Figure 2. IEA Tri-Partite Model of Curriculum

Recently, researchers have undertaken a description and analysis of curricular pathways and content trajectories (Kher, Schmidt, Houang, Cogan, Pearlman & Jiang, 2006; Kher, Schmidt, Houang & Wiley, 2005; Schiller, Schmidt, Picucci, Crumb, Muller & Houang, 2005). However, what has been absent from the recent work is the connection between the student opportunities to learn and the attained curriculum. The present study focuses on the connection between specific curricular pathways and student learning outcomes in mathematics. Specifically, we describe students' opportunities to learn mathematics content in two distinct school districts and determine if these opportunities to learn are linked with student performance in mathematics. We do not compare the two school districts but rather the intent is to understand the connections between the implemented and attained curriculum within each district. The

aim is to describe intra-district curriculum variations and their implications for student learning outcomes and curriculum policy.

Method

Instrumentation

Instruments used in our study are a subset of the 11 primary instruments used in the PROM/SE project. According to the National Advisory Committee members of PROM/SE, the assembled instruments have provided an unprecedented database of curriculum-sensitive baseline data on students, teachers, and districts. The data are organized to provide useful information to individual participating schools and districts.

In this section briefly described are:

- the conceptual basis of each instrument
- its relationship to other similar instruments
- the procedures and timeline for data collection

The measures used in this study were initially designed and used in the Third International Mathematics and Science Study (TIMSS). The instruments were subsequently revised and used in other national and international studies (see Schmidt & Cogan, 1996). Questions, and in some cases conceptual frameworks, were refined based on analyses done for these studies and were modified for the present study.

Student Assessments in Mathematics Grades 9 -12

Blueprints created by national experts, mathematicians and scientists, and mathematics and science educators from Michigan State University were used to design and construct the forms. This process took place during the six months preceding the assessment administration. Based on a recommendation from a team of

psychometricians experienced in the design and administration of large-scale assessments such as the ACT, NAEP, and TIMSS, a duplex design described by Bock and Mislevy (1988) was adopted.

Fifteen parallel forms were developed for mathematics in grades 9-12. Some items were taken from existing item pools that had been used for some state assessments, NAEP, and TIMSS. To provide international benchmarks and linkages a large number of items from the 1995 TIMSS were included in the assessments.

The PROM/SE mathematics assessment when analyzed as a group yield an indicator of students' mathematics performance on 27 distinct strands (topics). During the process of test development, a national group of mathematics experts identified these specific strands as being comprehensive of the mathematics content expected in the school curriculum.

Content Goals Survey Instrument for Teachers

Teachers responded to a list of topics that was exhaustive of school mathematics topics as represented by the TIMSS Curriculum Frameworks for Mathematics and Science (Survey of Mathematics and Science Opportunities, 1992a, 1992b). The present survey was a revision of the 1995 TIMSS version that demonstrated significant relationships with student learning in analyses of the 1995 TIMSS (Schmidt et al., 2001). The items in this instrument were designed for teachers to indicate the number of lessons they taught specific mathematics topics. Teachers were presented with a list 33 mathematics topics and for each topic they were supplied with response alternatives that focused on: a) how many class periods they taught the topic (0, 1, 2-5, 6-10, 11-15, and >15 class periods), b) student performance they most expected (knowledge, using

complex procedures, formulating problems, developing strategy and justifying) and, c) student performance occasionally expected. The instrument was available for teachers of grades 9-12 to complete as a web survey. A paper-and-pencil version was also available for teachers who had difficulty accessing the web.

Student Enrollment Data

To get the course enrollment history for an entire cohort of high school students we obtained enrollment data for all students in each high school beginning with the Fall, 1999 freshman cohort and ending with Spring, 2005. The data, in the form of an EXCEL spreadsheet contained student background information (student date of birth, expected graduation date, gender, ethnicity) and course-taking information (course title, semester and year taken or transferred, credit awarded, teacher name, and exit code). In addition to the EXCEL spreadsheets, we also obtained the official course catalogs for the years 2004 and 2005.

Participants

Data for the present study were obtained in the Spring of 2004 and 2005 from two geographically distant school districts. One school district (District A) with seven high schools is on the west coast of US and the other (District B) with one high school is from a Midwestern state. Both districts are located in mid-size central cities. District A has been engaged in district initiated curriculum reform for the past several years, whereas District B is part of an initiative focusing on comprehensive K-12 curriculum reform in mathematics and science.

Data from all high school students and their mathematics teachers were obtained. Participant characteristics are presented in Table 1.

Table 1: Study Participants

Data Source	District A	District B
High School Mathematics Teachers	75	32
Number of Participating High Schools	7	1 ¹
Students (Grades 9-12)	4,956	1075
Student/Teacher Ratio	12.0 to 22.3	19.1
Free/Reduced Lunch Eligible	6.4% to 40%	11%
Percent Minority enrollment	15% to 42%	15%
School Characteristics	Mid-size ² Central City	Mid-size Central City

Procedures

During Spring, 2004, the mathematics teachers in District B responded to the web-based version of the Teacher Content Goals Survey to determine content coverage in each mathematics course taught. Teachers in District A were administered a paper-pencil version of the instrument. Teachers indicated the extent to which they taught thirty-three different mathematics topics. These topics were collapsed into ten broad mathematics areas (Appendix A).

The PROM/SE mathematics assessments were administered to District A students in Spring, 2005 and to District B students in Spring, 2004. The assessments were administered in a group setting during the course of one class period. Thus, District B received the original and District A the revised version of the assessment. The items on both assessments were equated for comparability. In addition, students' course enrollment records were obtained.

¹ There is only one high school in District B

² Based on NCES classification

Analysis

In the present study, three distinct data sources have been linked; teacher content coverage, student performance and student enrollment. Separate analyses related to all three data sources were conducted. Inferential methods of analysis are not deemed necessary as the entire population of students and have been surveyed. Preliminary analyses based on the linked data sets have been conducted.

To determine performance on the broad mathematics topics by courses taken, students were grouped according to the course and class they were enrolled in when they completed assessments. Courses taken also allowed us to link teacher content coverage with course-taking and mathematics performance.

For each district, teacher content coverage data were grouped by course and summarized. Using a formula based on total allocation of teaching time over the entire academic year, the responses based on class periods were converted to percentage of teaching time for each topic. Percentages of teaching time in Ten Broad Mathematics Areas were computed by collapsing relevant topic categories from the Teacher Content Goals Survey.

The PROM/SE assessments provide aggregated information. Thus, students' scores are aggregated at the classroom level. The scores can be further aggregated to the course, school or district level. The results presented here are based on course level aggregation.

Mathematics course sequences were identified from enrollment data. These sequences provide descriptive information about course-taking patterns. Precedence and adjacency matrices where the rows and columns of the matrix represent the various

mathematics courses offered at each school district and entries in the matrix represent the number of students taking the course and the order of course-taking. Summaries of these matrices provide a model of course-taking patterns and empirical evidence of the extent of curricular tracking.

Results

Teacher Content Coverage

Displays 1 and 2 are based on teachers' reports of the percentage of time devoted to mathematics topics in ***District A*** and ***District B***, respectively. If more than 4 teachers reported teaching the same course a summary was generated. In addition to the average percent time for each of the ten broad content areas (Appendix A), we also determined the standard deviation, minimum and maximum reported time. The minimum and maximum percentage of time reported by the teachers is presented in Table 2 for ***District A*** and Table 3 for ***District B***. Displays 1 and 2 provides information about typical content coverage in the mathematics courses and Tables 2 and 3 depict the variation that exists within the district in terms of content coverage.

In ***District A*** (Table 2), it is evident that in the AP Statistics course the range of content coverage in the topic area of *Data and Statistics* from 34% to about 92 % and *Advanced Algebra* coverage ranges from approximately 3% to more than 25%. Teachers who teach Integrated Algebra/Geometry 2 vary considerably in terms of their coverage of ***Basic Geometry*** where the coverage ranges from 7.5% to 38.5%. In at least one of the classrooms, teachers have no coverage of *Number* (basic and transition) whereas in another classroom where the teacher was teaching the same course, almost one-third of the time was spent on *Number*.

In **District B** (Table 3), the treatment of *Basic Geometry* in both the Integrated Math Beginning and Integrated Math Advanced differs considerably from classroom to classroom (0% to 35.7% and 0% to 24.7%, respectively). The teachers who teach the Advanced Algebra course in **District B**, report a coverage range from approximately 32% to 45% for the area of *Advanced Algebra*.

Student Enrollment and Course-Taking

Student enrollment information is presented in Tables 4 and 5. About half the students in **District A** begin their ninth grade mathematics coursework in the Honors Algebra/Geometry 2 class and about a quarter of the students take Integrated Algebra/Geometry 1. In twelfth grade about 22% of the students are enrolled in AP Calculus AB and a similar number (23%) are enrolled in AP statistics (Table 4).

Students in **District B** have many more course offerings and thus, considerable variation in opportunities to learn. Courses include General Math, Life Skills Math, Integrated Math and courses at the Math Science Center (MSC) for accelerated coursework. About 38% of the ninth grade students in District B take Integrated Math 1 and about 17% take Integrated Math 2. In twelfth grade, 22% of the students take MSC-Advanced Geometry, 19% of the students enrolled in AP Statistics and 15% in AP Calculus AB (Table 5).

Course taking patterns for **District A** are presented in Figures 3 and 4. Courses depicted in green boxes represent entry points. Arrow directions indicate course-taking sequences. These figures illustrate the presence of tracking in **District A**. Three distinct tracks appear to be most frequent (Figure 3) with the starting points of Algebra 1-2, Integrated Algebra/Geometry 1, and Honors Algebra/Geometry 2. Figure two includes

all the course taking patterns and here the movement between tracks becomes apparent. This movement may reflect placement adjustments made by guidance counselors or changes in student aspirations.

Course taking patterns for **District B** are depicted in Figures 5 and 6. As District B has a set of courses offered through the Math Science Center (MSC), Figure 5 depicts the course-taking patterns for non-MSC courses, whereas, Figure 6 focuses on MSC coursework. Tracking is also apparent in **District B**. Here the starting points appear to be: Integrated Math 1 and Algebra 1 for the non-MSC group and MSC Accelerated Math and MSC Fundamentals 1-2 for the MSC group. Figure 5 also illustrates the placement-adjustment phenomenon. In Figure 6, courses in blue boxes are non-MSC courses. Some students start with MSC courses but are channeled to the Integrated Math sequence.

Connecting Course-taking and Performance

Student groups were identified based on their mathematics course enrollment when the PROM/SE assessments were administered. Student performance on TIMSS high school mathematics literacy items was extracted from the overall PROM/SE assessments. Student performance by course enrollment is presented in Table 6 and 7 for **Districts A and B**, respectively. Courses identified in the tables are ones that had student performance data for at least 9 students. In both districts higher levels of course taking are associated with a higher TIMSS literacy score.

Conclusions and Implications

Our findings support Schmidt's (2002) and Kher et. al's (2005; 2006) contention that curriculum differentiation exists at the high school level. The data suggest that

different content trajectories offer very different opportunities to learn within and between school districts. Teacher content coverage data also point to variability in the implemented curriculum which also leads to differences in students' opportunity to learn. Our data further suggests that content trajectories are linked to levels of student performance in mathematics

Researchers have maintained that curriculum differentiation can potentially create inequalities in students' opportunities to learn. Tracking or ability grouping practices lead to instructional differences (Hallinan & Kubitschek, 1999; NRC, 2002). Early differential placement can channel students away from rigorous programs of study and such curriculum differentiation has several attendant consequences. High ability groups receive more complex instructional materials (Metz, 1987; Oakes, 1990; Page, 1987, 1991), have more stimulating interactions with teachers (Gamoran & Mare, 1989), and take higher level of courses (Advanced/Honors vs. Basic/Very Basic) which are related to achievement (Hallinan, 1996).

In the 80s and 90s de-tracking the K-12 educational system was an imperative because it was believed to create inequity in educational opportunity. Formal tracking may have been replaced by differential placement practices which may have the same affect. The variety of course options available to fulfill graduation requirements are bewildering for students who have no knowledge about the implications of their course choices (Rice, 1997). This confusion is shared by parents who may not realize the full impact of curriculum differentiation and placement on future academic choices (LeTendre et al., 2003).

It is clear that the courses students take to fulfill requirements matter in terms of their learning outcomes and the available future academic choices. With the proliferation of course offerings, high school students' course-taking choices do not reflect clear content standards and appear arbitrary, and misguided. Our findings confirm the conclusion reached by LeTendre et al. (2003) that replacing tracking with a "hodgepodge of lanes, specializations and 'choices'...is essentially still tracking students, and such a system may only increase educational inequity" (p.81).

Lack of clear standards coupled with a "smorgasbord" of choices creates a set of artificial tracks in the curriculum that adversely affect mathematical literacy, and also limit students' future educational and career opportunities. The recent report from the National Academies (2006), *Rising Above the Gathering Storm*, has seriously questioned the future of U.S.'s competitiveness in the areas of science and technology. The report has underscored the need to improve K-12 mathematics and science education to support an increase in both the talent and innovation capabilities of the U.S. workforce. We believe that a curriculum that is focused, coherent, challenging, and expected of **all** students is crucial for the U.S. in, "renewing its workforce with adequate rigor and foresight" (p.1)³

³ National Mathematics Advisory Panel: Preliminary Report (January, 2007)

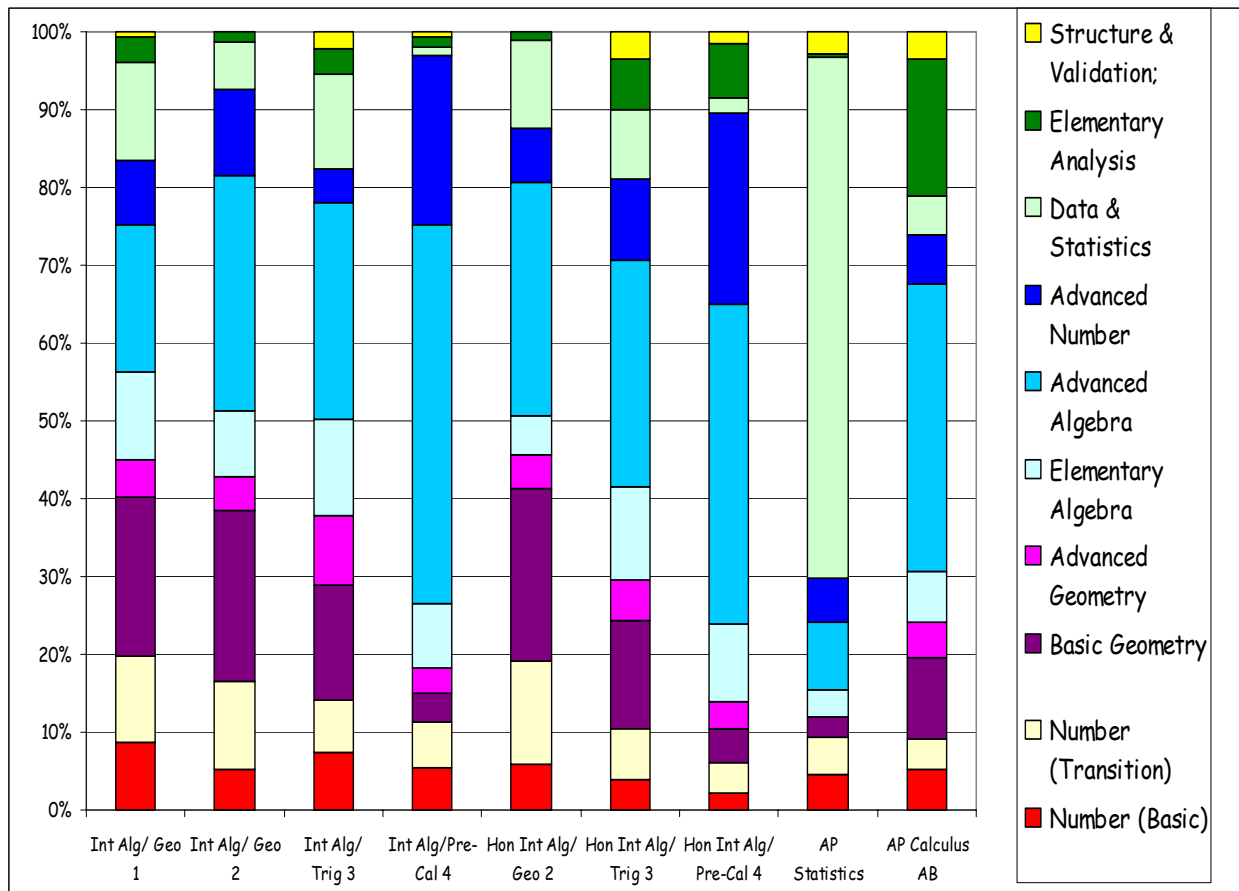
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Display 1. Average Percent of Teaching Time by Course Type: District A



Display 2. Average Percent Teaching Time by Course Type: District B

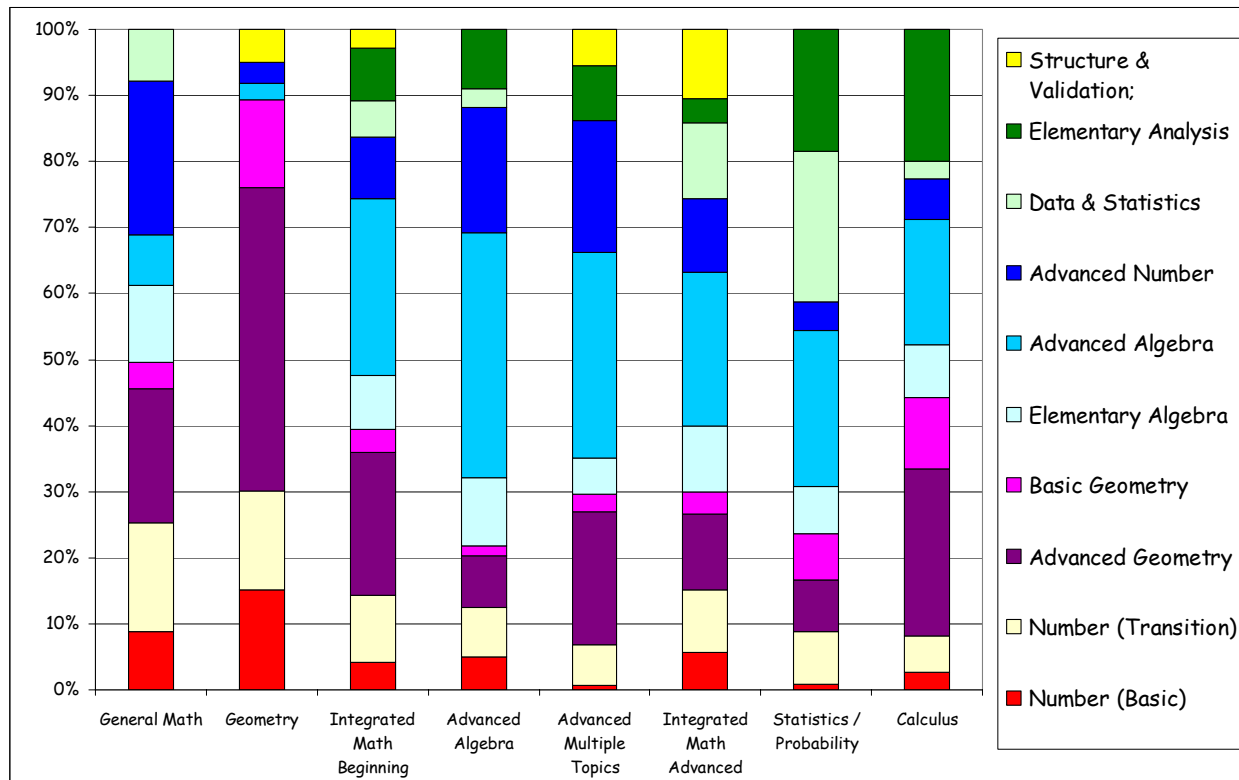


Table 2. Minimum and Maximum Percent of Teaching Time by Course Type District A

Ten Board Mathematics Areas	High School Course																	
	Int Alg/ Geo 1		Int Alg/ Geo 2		Int Alg/ Trig 3		Int Alg/Pre- Cal 4		Hon Int Alg/ Geo 2		Hon Int Alg/ Trig 3		Hon Int Alg/ Pre-Cal 4		AP Statistics		AP Calculus AB	
	Min %	Max %	Min %	Max %	Min %	Max %	Min %	Max %	Min %	Max %	Min %	Max %	Min %	Max %	Min %	Max %	Min %	Max %
Structure & Validation	0.0	4.2	0.0	0.6	0.0	4.6	0.0	1.6	0.0	0.0	0.0	14.7	0.0	5.8	0.0	10.3	0.0	6.3
Elementary Analysis	0.0	6.7	0.0	7.7	0.0	7.5	0.0	3.3	0.0	5.6	1.8	12.5	0.0	22.2	0.0	2.4	3.3	32.8
Data & Statistics	5.6	18.5	0.0	16.7	6.9	15.3	0.0	2.1	2.2	25.0	1.5	23.1	0.0	4.0	34.2	92.9	0.0	15.9
Advanced Number	3.5	13.8	1.4	17.2	0.9	8.2	16.1	25.0	2.6	13.4	0.0	13.2	11.8	48.8	0.0	11.9	3.1	15.2
Advanced Algebra	9.2	34.2	19.2	50.0	19.2	33.9	30.4	67.7	17.5	48.6	3.8	29.8	27.9	51.6	2.6	26.3	17.2	63.2
Elementary Algebra	6.2	16.5	2.2	16.4	2.7	19.3	2.8	11.4	0.0	12.7	3.5	42.1	0.0	19.7	0.0	11.8	0.0	18.8
Advanced Geometry	0.0	10.8	1.1	7.7	6.6	12.3	0.0	11.1	0.9	11.6	2.1	28.6	0.0	9.6	0.0	0.0	0.0	9.8
Basic Geometry	12.3	33.3	7.5	38.5	10.1	19.7	1.0	7.9	9.9	33.3	3.7	17.9	2.0	9.4	0.0	10.3	0.0	26.7
Number (Transition)	0.7	16.7	0.0	20.2	1.8	13.7	3.2	8.8	2.6	25.0	1.1	19.2	0.0	9.8	0.0	11.8	0.0	12.1
Number (Basic)	0.0	13.8	0.0	11.5	1.1	16.4	0.0	14.2	0.0	17.4	0.0	9.5	0.0	5.6	1.3	7.4	0.0	9.1

Table 3. Minimum and Maximum Percent of Teaching Time by Course Type District B⁴

Ten Broad Mathematics Areas	General Math			Geometry			Integrated Math Beginning			Advanced Algebra			Advanced Multiple Topics			Integrated Math Advanced			Statistics / Probability			Calculus		
	Ave %	Min %	Max %	Ave %	Min %	Max %	Ave %	Min %	Max %	Ave %	Min %	Max %	Ave %	Min %	Max %	Ave %	Min %	Max %	Ave %	Min %	Max %	Ave %	Min %	Max %
Structure & Validation	0.0			5.0			2.9	0.0	17.1	0.0	0.0	0.0	5.5			10.6	0.0	24.7	0.0			0.0		
Elementary Analysis	0.0			0.0			8.0	0.0	28.6	9.0	6.3	13.6	8.3			3.5	0.0	9.9	18.4			20.0		
Data & Statistics	7.8			0.0			5.5	0.0	14.3	2.8	0.0	7.2	0.0			11.6	5.4	19.8	22.8			2.7		
Advanced Number	23.3			3.1			9.4	0.0	26.3	19.0	15.3	21.3	20.0			11.0	5.2	16.2	4.4			6.1		
Advanced Algebra	7.8			2.5			26.7	8.8	41.7	37.1	32.4	45.0	31.0			23.3	16.5	27.0	23.7			18.9		
Elementary Algebra	11.7			0.0			8.1	0.0	17.6	10.3	10.0	10.8	5.5			10.0	4.9	12.4	7.0			8.1		
Advanced Geometry	3.9			13.2			3.6	0.0	6.3	1.5	0.0	3.6	2.8			3.4	0.0	5.4	7.0			10.7		
Basic Geometry	20.4			45.9			21.6	0.0	35.7	7.8	5.0	11.7	20.0			11.5	0.0	24.7	7.9			25.4		
Number (Transition)	16.5			15.1			10.2	6.3	18.6	7.4	7.2	7.6	6.2			9.5	0.0	19.8	7.9			5.4		
Number (Basic)	8.7			15.1			4.1	0.0	8.8	5.0	3.8	6.8	0.7			5.6	0.0	13.4	0.9			2.7		

⁴ Minimum and Maximum are not provided when n,3

Table 4:Percentage of Students in Each Grade Taking Different Courses (District A)

Courses	Grades			
	9	10	11	12
MATH TUTORIAL			0.4	0.1
MATH SUPPORT			0.1	0.0
INTEG MATH 1	1.1	0.6	0.3	0.0
INTEG MATH 2	0.1	0.0	0.1	0.1
INTEG MATH 2A	0.1	0.0	0.2	0.1
INTG ALG/GEO1	25.6	6.0	2.6	0.7
INTG ALG/GEO2	5.8	20.8	8.0	3.0
INT ALG/TRIG3	0.4	8.3	18.0	9.0
INT AL/PRCAL4	0.0	0.3	10.3	15.5
HON ALG/GEO 2	49.4	6.6	0.8	0.4
HON ALG/TRIG3	15.3	41.2	8.7	2.5
HON AL/PRCAL4	1.2	13.8	31.5	10.4
AP CALCULUS AB	0.7	1.7	11.9	22.0
AP CALCULUS BC	0.1	0.1	0.6	2.3
AP STATISTICS		0.7	2.3	23.7
INTRO/CALCULUS			0.8	2.5
ADV CALCULUS			0.4	2.6
IB MATH STUDIES			1.9	2.9
IB MATH HL				1.6
IB MATH MTHD SL			1.0	0.6
Number in Each Grade taking Math	1218	1175	1065	692

Most frequently taken course in the grade

Second most frequently taken course in the grade

Third most frequently taken course in the grade

Table 5: Percentage of Students in Each Grade Taking Different Courses *District B*

Courses				
	9th Grade	10th Grade	11th Grade	12th Grade
ALGEBRA	0.5	0.3	0.3	0.0
ALGEBRA 1	1.6	0.9	0.3	0.0
APPLIED MATH	1.6	0.6	0.0	0.0
BASIC MATH	0.0	0.0	0.3	0.0
CONSUMER MATH	0.5	0.0	0.0	0.0
GENERAL MATH	0.5	0.0	0.0	0.0
GEOMETRY	16.1	1.7	0.7	
MATH	1.3		0.7	
MATH 2		0.3		
INTEG MATH 2	17.5	30.4	11.9	5.1
INTEG MATH 1	38.1	9.4	3.1	1.3
INTEG MATH 3	3.2	16.8	14.3	3.2
INTEG MATH 4		7.4	11.2	7.0
INTEG MATH 5	0.3	1.1	8.5	5.1
CORRECTIVE MATH	2.4	2.3	0.7	0.6
MSC-ACC MATH	2.9	2.8	0.0	
MSC-ADV STAT			3.7	2.5
MSC-CALCULUS BC			3.1	3.8
MSC-MULTI VARIATE				2.5
ALGEBRA 2	4.0	11.6	3.4	0.6
AP CALC AB			3.1	15.3
AP CALCULUS BC			0.7	3.8
AP STATISTICS			3.4	19.1
PRECAL,STAT,TRIG			6.8	4.5
TRIG/VECTORS	0.3	5.4	9.5	3.2
MSC-CALCULUS AB		0.0	3.7	
MSC-PRECAL REAS			19.3	
ADV STATISTICS		0.6	0.0	
MATHEMATICAL COMPUTATION		1.4	0.0	1.3
APPL MATH 2	1.3	0.9	0.0	
MATH APPLICATION			1.4	
MSC-ADV GEOMETRY			1.0	21.6
MSC-DISCRETE MATH			1.0	8.7
MSC-FUNDMATH1/2	3.4			
MSC-FUND MATH 3/4		3.4		
MSC-FND MTH3	0.3			
MSC-FUND MATH 4		0.3		
MSC-AP STAT			1.4	1.3
MSC-AP STAT2			0.3	1.3
STATISTICS				0.6
MATH1			0.3	
FUND. OF ALGEBRA	0.3			
BASIC ALGEBRA		0.3		
MSC AD COMPUTERS	0.5			
MSC-AP PRECALC		0.3		
GEOMETRY 1	0.3			

Courses (Continued)	9th Grade	10th Grade	11th Grade	12th Grade
GEOMETRY 2	0.3			
ALGEBRA 3		0.3		
BAS ALG 2	0.3			
BASIC ALG 1	0.3			
GEOMETRY-HOME SCHOOL	0.3			
PRINCIPLES OF MATH	0.5			
CONT MATH 1	0.3			
CLASS LAB		0.3		
MATH APP2		0.3		
APP MATH III			0.3	
ADV ALGEBRA		0.3		
ALGEBRA ADVANCED HONORS	0.3			
HONORS TRIG		0.3		
ALGEBRA 1 A	0.3			
ALGEBRA 1B	0.3			
ADV FUNCTION				0.6
STAT/TRG	0.3			
ADVANCED ALGEBRA		0.3		
PRECALC			0.3	
LIFE MATH	0.3			
LIFE MATH SKILLS		0.3	0.3	
GEOMETRY UNIFIED			0.3	
Number Taking Course	378	352	294	157
Most frequently taken course in the grade				
Second most frequently taken course in the grade				
Third most frequently taken course in the grade				

Figure 3. Math Course-taking Patterns: District A

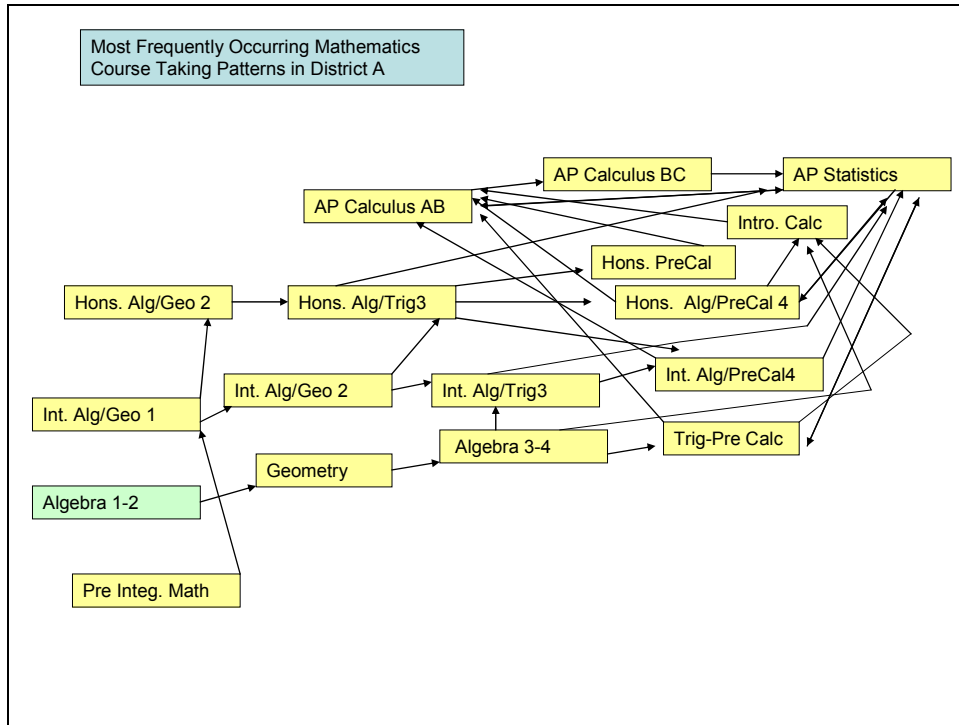


Figure 4. Math Course-taking Patterns: District A

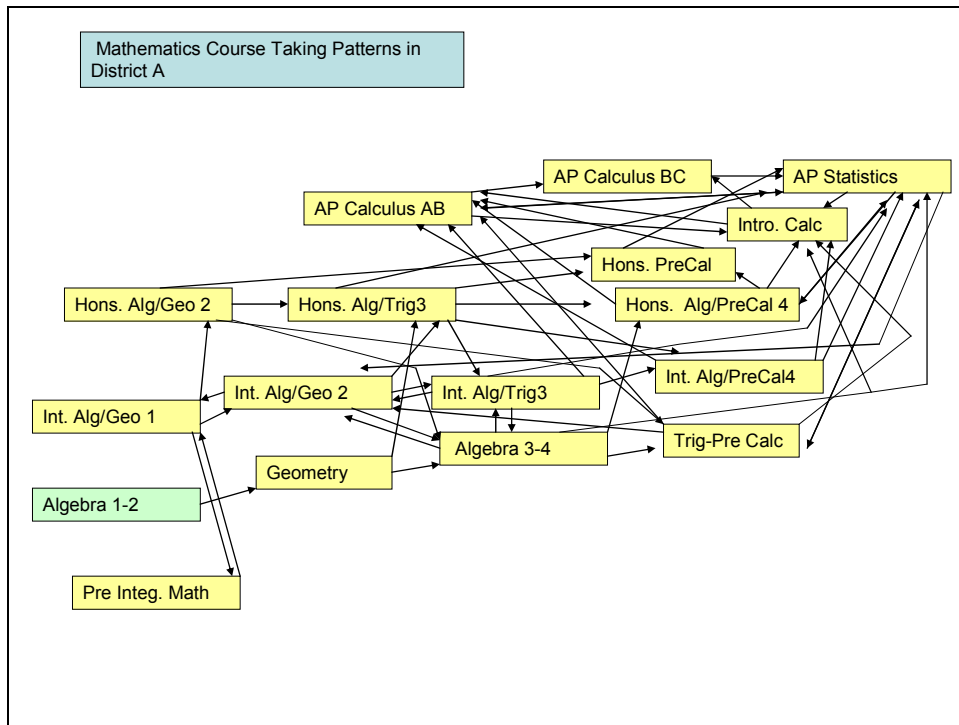


Figure 5. Math Course-taking Patterns: District B

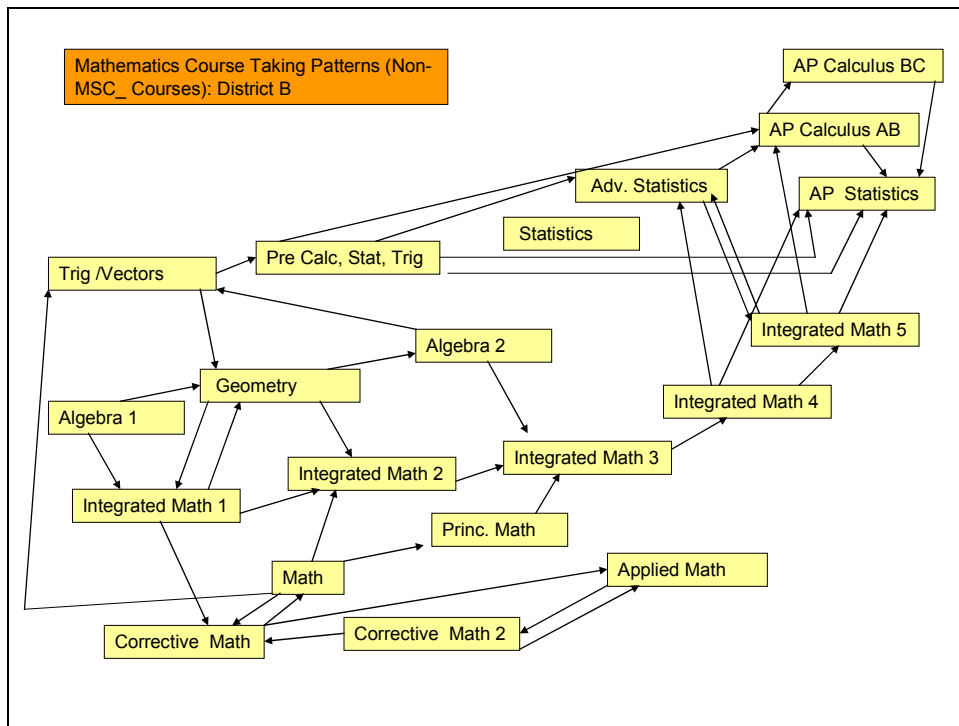


Figure 6. Math Course-taking Patterns: District B

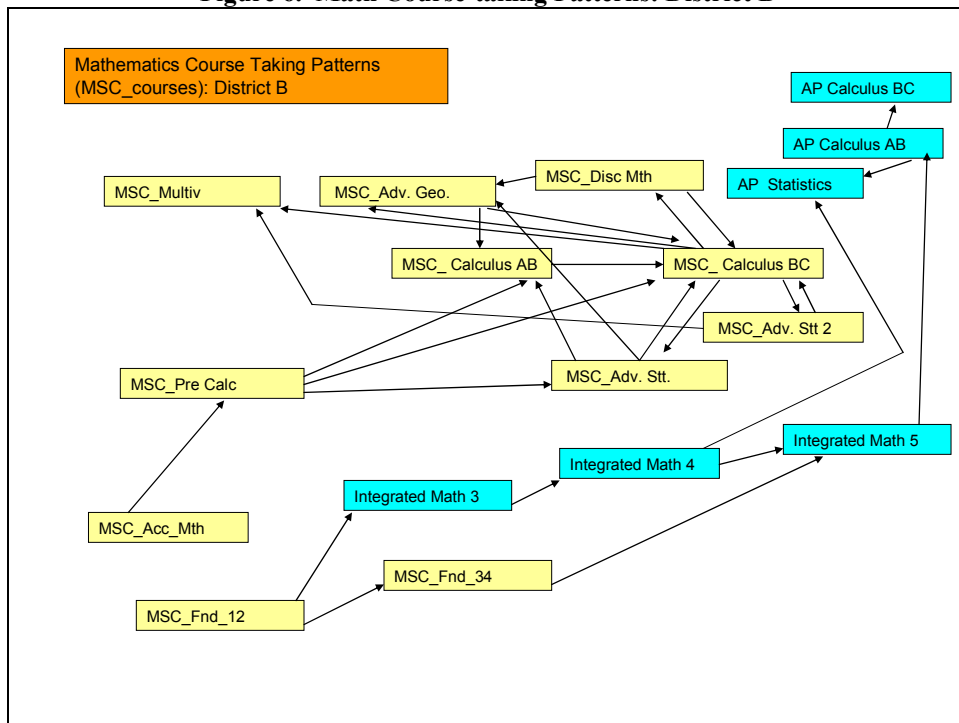


Table 6: Student Performance on TIMSS Literacy Items by Course: District A		
Course Taken	Percent Correct	
Adv Calculus	93	
Intro/Calculus	88	
AP Calculus AB	85	
Hon Alg/PrcaL4	81	
AP Statistics	78	
AP Calculus BC	77	
Hon Alg/Trig3	75	
Int AlgPrcal4	60	
Int Alg/Trig3	55	

Table 7: Student Performance on TIMSS Literacy Items by Course: District B		
Course Taken	Percent Correct	
Calculus	80	
Statistics/Probability	75	
Integrated Math Advanced	64	
Algebra II	64	
General Math	59	
Integrated Math Beginning	50	

Appendix A
Ten Broad Mathematics Areas⁵

Number (Basic)

- 1) Whole Numbers** – place value & numeration; ordering & comparing; operations & properties of operations
- 8) Estimation & Number Sense** – estimating quantity & size; rounding; significant figures; estimating computations; exponents & orders of magnitude
- 9) Units** – standard non-standard units (e.g., liters, paper clips); measures of area, time, temperature, money, etc.; miles/hour, miles/gal., etc.; making measurements

Number (Transition)

- 2) Fractions & Decimals** – meaning, representation, uses, computations; equivalence, conversions & ordering
- 11) Estimation & Measurement Errors** – measurement estimation; precision & accuracy of measurements
- 17) Proportionality Concepts** – direct & inverse proportions; ratio
- 18) Proportionality Problems** – scales, maps, models, proportional equations

Basic Geometry

- 10) Perimeter, Area, & Volume** – computations, formulas, & properties (including surface area)
- 12) Coordinates & Lines** – number lines, graphs; segments, rays; angles; equations of lines; parallelism & perpendicularity
- 13) Polygons & Circles** – classification, formulae, properties, & theorems of circles, triangles, & other polygons
- 15) Transformations** – patterns, tessellations, friezes; symmetry, rotation, & reflections

Advanced Geometry

- 14) Three Dimensional** – spatial visualization; 3-D coordinate systems; vectors; constructions with straightedge & compass
- 16) Congruence & Similarity** – properties of congruence & similarity

Elementary Algebra

- 22) Expressions & Simple Equations** – representing numerical situations; informal solutions; factorization & simplification; substitution into formulas
- 23) Linear Equations & Inequalities** – formal solutions of linear equations & inequalities

Advanced Algebra

- 19) Slope & Trigonometry** – slope & interpolation; sines & cosines
- 20) Patterns and Relations** – number patterns, mathematical relations and their properties
- 21) Functions** – types and properties of functions, operation on functions, relationship of functions & equations, interpretation of function graphs

⁵ The numbers reflect individual topic numbers that were aggregated to create the ten broad areas.

- 24) Quadratic & Polynomial Equations & Inequalities** – formal solutions for such equations, inequalities and their graphical representation
- 25) Logarithmic & Trigonometric Equations** – solution methods for such equations (e.g., successive approximation); exponential equations
- 26) Systems of Equations & Inequalities** – properties & solutions of such systems including matrix algebra

Advanced Number

- 3) Negative, Rational, & Real Numbers** – meanings, operations, & properties
- 4) Number Bases** – binary arithmetic; bases other than ten; meaning of logarithms
- 5) Exponents, Roots, & Radicals** – properties & operations; relations between roots, radicals, & exponents
- 6) Complex Numbers** – concepts & properties; algebraic & trigonometric forms of complex numbers
- 7) Number Theory** – primes & factorization; systematic counting; permutations & combinations

Data & Statistics

- 27) Representing & Interpreting Data** – tables, charts, graphs; mean, median, mode, & sampling
- 28) Probability & Uncertainty** – concepts of “more likely” & “less likely”; contingency tables, confidence intervals, hypothesis testing

Elementary Analysis

- 29) Infinite Processes** – arithmetic & geometric sequences & series; limits & convergence of series
- 30) Change** – growth & decay; differentiation & integration; differential equations; partial differentiation

Structure & Validation

- 31) Validation & Justification** – logic; Boolean algebra & truth tables; inference schemes; proofs
- 32) Structuring & Abstracting** – sets & notation; equivalence relations; groups; vector spaces; axiomatic systems; isomorphism; homomorphism

