Session Title:
An Adaptive Model for Supporting Shifts in Secondary Precalculus Instruction and Student Learning

MSP Project Name:
Project Pathways: A Math and Science Partnership Program for Arizona Targeted Project Track

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Project Session

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Summary:
The Pathways Precalculus Professional Development Model (P^3DM) includes focused workshops for teachers, student curriculum and instructor support materials. Our research to scale Pathways to all teachers within a school and district have revealed new challenges, while supporting past results of the importance of sustained interventions to develop teacher's Mathematical Knowledge for Teaching (MKT). This session will engage participants in an activity to illustrate the process by which teachers acquire MKT, and how a teacher's emerging pedagogical conceptions and knowledge of the content impacts her instructional choices and classroom interactions. We will also report on the wide variation we have observed in teachers' knowledge, school leadership, and parental support, and how we are responding to these variations in scaling Pathways.

Section 1: Questions framing the session:
- What do we mean by mathematical content knowledge for teaching precalculus?
- What supports are necessary and effective in realizing shifts in secondary teachers' mathematical content knowledge for teaching, and how do these shifts impact their teaching?
- What is the process by which the *Pathways Professional Development Model* (*P^3DM)* is being adapted?
- What new insights have been gained when attempting to scale Pathways to all mathematics teachers within a district or school?

Section 2: Conceptual framework:
Project Pathways Phase II has a primary goal to affect secondary precalculus teachers' instructional practices so that teachers are able to: i) support students in constructing deep understandings and rich connections among central ideas of precalculus; ii) support students in developing problem solving abilities that entail *ways of thinking* and *ways of approaching* complex tasks that include accessing and using key concepts as needed to

solve novel problems; iii) interpret and act on student thinking when teaching; iv) improve student learning; and v) reflect on student thinking and learning to improve teaching. This definition of effective teaching emerged from studying teachers in relation to their knowledge, curriculum, and classroom practices during our Phase I research. Our early findings revealed the nature of mathematical content knowledge for teaching specific precalculus content, and uncovered numerous complexities in designing professional development and resources that lead to observable and significant shifts in a teacher's instructional practices.

We base our research on the Silverman and Thompson (2008) framework . The framework characterizes the process by which a teacher's mathematical knowledge for teaching (MKT) may develop. According to this framework, the teacher first develops a significant mathematical understanding by engaging with tasks that support the teacher in constructing his/her personal meanings and connections. As this knowledge becomes more and more connected and useful in the mind of the teacher, the teacher may eventually realize its value for his/her students. As the teacher is supported in further reflection and attention to student thinking the teacher develops ideas of how to support her students in acquiring the ways of thinking and meanings inherent in learning, understanding and using a concept to solve novel problems. Also, as the teacher engages in on-going reflection, he/she continues to develop new ways of thinking and new approaches to teaching to help students develop these ways of thinking.

Pathways researchers have investigated the processes by which teachers acquire MKT for each major idea of precalculus level mathematics, beginning with proportionality and its relationship to constant rate of change, to function and function composition, exponential growth, and trigonometric functions in both the unit circle and triangle contexts. The frameworks that have emerged from these studies (e.g., Oehrtman, Carlson & Thompson, 2002; Strom, 2008; Moore, 2009; Moore & Carlson, 2012) have informed the design of the interventions we developed to support teachers in first acquiring the mathematical understandings and connections of central ideas at the precalculus level.

Our work to help teachers appreciate the usefulness of their students acquiring deep and connected understandings revealed that shifts in teachers' knowledge of the mathematics of the courses they teach may not lead to noticeable shifts in teaching and student learning. These findings are consistent with the Silverman and Thompson (2008) framework, as they reported that even when secondary mathematics teachers do transform their personal understandings of the mathematics, this transformation may not result in subsequent transformations in their pedagogical conceptualizations. As Silverman and Thompson described, a key aspect of developing MKT is transforming one's personal meanings in a way that considers how his/her students might develop compatible meanings.

Our study of secondary teachers revealed that the process of realizing dramatic gains in a teacher's pedagogical conceptions and teaching effectiveness was met by many obstacles, including the daunting task of revising lessons for an entire year, the requirement that students complete a procedurally focused district exam as the primary measure of their

learning, and student and parent resistance to conceptual learning. This led to our continued refinement of the *Pathways Professional Development Model* to include student curriculum and instructor support materials to assure that teachers are using tasks that will realize advances in teachers' pedagogical conceptions, their students' thinking and the quality of their interactions with students. As with all Pathways tools, these materials and tools are under an iterative process of research and adaptation, with adaptations based on research of their effectiveness when used by teachers in their classrooms.

Section 3: Explanatory framework:

Our research of Pathways teachers is providing novel insights about the process by which secondary mathematics teachers become more effective teachers as previously defined in this proposal. We are learning that teachers are capable of making rapid gains in their teaching practice that result in significant gains in student learning provided they are adequately supported in: i) constructing deep understandings and rich connections among central ideas of precalculus; ii) developing problem solving abilities that entail ways of thinking and ways of approaching complex tasks that include accessing and using key concepts as needed to solve novel problems; iii) considering and using student thinking to inform their teaching decisions; iv) engaging in regular *reflection on student thinking and learning* in relation to their teaching in order to understand how students develop particular meanings and understandings. A study of 12 teachers using P^3DM revealed highly significant gains in their students' mean scores on the Precalculus Concept Assessment (PCA). The mean PCA gain score for precalculus students taught by P^3DM trained teachers ranged from 5.5 to 9, with a mean post PCA score of 15.5 (out of 25) as compared to a mean post PCA score of 10.2 (out of 25) from over 40 classes of college level precalculus courses. Our early data also supports that students of P^3DM teachers are well prepared to succeed in calculus. These findings support that coherent research-based curriculum designed to support student construction of meaning can be broadly effective in improving student learning.

Our efforts to scale Pathways interventions to all secondary mathematics teachers within a district have further revealed a wide variation in teacher knowledge that needs to be addressed. In instances when teachers attended only a 3-5 day summer workshop prior to using Pathways materials we observed that some teachers are not ready to use the Pathways materials because their MKT (as defined by Silverman and Thompson to include both their content knowledge and pedagogical conceptions) did not develop to the point that these teachers were able to provide coherent explanations, consider and use student thinking, and adequately respond to student questions. In instances where teachers' MKT knowledge is extremely weak, student understanding is not supported, resulting in some students expressing high frustration to their parents. Our findings reveal that weak implementations in only a few classrooms can lead to strong reactions by parents and students that need to be managed through school leaders (i.e., principals, department chairs) and Pathways leaders. These recent findings have further revealed the important role of school administrators in supporting teachers in making the shift to Pathways philosophies, materials and teaching.

Section 4: Discussion:
Pathways findings have revealed both the importance and complexity of developing teachers' MKT, including transforming teacher conceptions (both in content and pedagogy) so that they implement lessons that engage students in meaningful constructions that support student development of foundational understandings. The process of transforming all mathematics teachers within a school presents challenges, especially in situations where a teacher's MKT is extremely weak, or when a teacher is not invested in the Pathways philosophies. If higher education project members have failed to develop a genuine intellectual partnership with school administrators and teacher leaders, strong negative reactions from only a few teachers and/or parents may lead to schools abandoning work to shift toward conceptual mathematics teaching. Collectively, these findings are leading to modifications in Pathways interventions that attempt to address the complexities of developing teachers' MKT. Also, these findings are providing knowledge about the support needed from district and school administrators to realize and sustain shifts in teachers' practice that align with the Pathways vision.

Section 5: How will you structure this session? What is your plan for participant interaction?
The session will begin by having the participants discuss what MKT for teaching ideas of proportionality, constant rate of change, and linearity would entail. The presenter will solicit ideas from the audience. The participants will then complete a Pathways in-class activity that supports students in understanding these ideas and applying their knowledge of how they are connected. The presenter will then revisit the discussion of what MKT is needed to implement this task effectively in a classroom. The session will continue by reporting how Pathways materials support teachers in acquiring MKT for teaching these ideas. We will then share and discuss classroom data that illustrates how variations in MKT impact instruction. The session will conclude with discussions of the role of school administrators in supporting teachers to implement Pathways materials, and in addressing problems that arise when new demands are placed on both teachers and students.