Session Title:
Interdisciplinary Perspectives in a Teacher Development Program

MSP Project Name:
Poincaré Institute: A Partnership for Mathematics Education

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Project Session

Strand 1

Summary:
We describe the Poincaré Institute's efforts to develop interdisciplinary graduate level courses, along with school-based support, for grades 5 to 9 teachers. In the courses, teachers examine the content of middle school mathematics in depth, going beyond what they teach, and from the perspectives of mathematics, mathematics education, and physics, including investigations of students' learning and reasoning. We mention previous collaboration leading to the project and on the evolution of our shared vision towards the improvement of teaching and learning. We illustrate our progress by examining how the first course was structured and later adjusted on the basis of what we have learned from the first cohort of 60 participant teachers from nine New England School Districts.

Section 1: Questions framing the session:

What forms of productive **interdisciplinary collaboration** among the MSP contributors can we devise that go beyond having mathematicians *doing the math*, educators *doing the pedagogy* and scientists *doing the science*? [Short answer: All course materials are considered collective products that undergo discussion, reformulation, editing from all disciplines (and roles in the project). Joint writing of notes and challenge questions; interdisciplinary editorial board; interdisciplinary online coordination; interdisciplinary site visits.]

What sorts of **adjustments** or **concessions** does an interdisciplinary approach demand with regard to how content is presented or problems are designed? [(1) Examples precede definitions; (2) functions first introduced as dependencies among quantities (and sets of numbers) and later as mappings from elements of a domain to elements of a co-domain; (3) proof and rigor given more attention than normal. (4) Modeling given a prominent role.]

How do we expect teachers to change their approaches to mathematics teaching as a result of the interdisciplinary approach the Institute offers? [We hope they will always (1) be looking for overarching mathematical issues and mathematical properties and structures; (2) attempting to compare how the text frames issues versus how students make sense of the same issues; (3) taking into account the peculiar ways in which the quantities chosen may influence how one teaches or learns about a topic (e.g. division as sharing, partitioning, creating an intensive quantity, etc.)]

Section 2: Conceptual framework:
The Poincaré Institute's interdisciplinary partnership arose out of more than a decade of prior collaboration across four NSF funded classroom investigations of algebra in the early grades (earlyalgebra.org), the preparation of teachers and researchers at Tufts graduate level programs in Mathematics Education, and the commitment, shared by faculty from the Education, Mathematics, and Physics departments at Tufts University and researchers from TERC to improve mathematics teaching and learning at all levels. We aimed at a program that would equally consider deep mathematical content, student and teachers ways of thinking and learning, and mathematical reasoning about physical quantities and modeling everyday and scientific phenomena.
Since the early planning stages, three ideas have played fundamental roles.

1. The concept of **function** offers a key for reinterpreting, within a single coherent framework, a wide range of topics in the K-12 curriculum (including the operations of arithmetic, fractions, divisibility, ratio and proportion, algebra, equations, graphs, etc.)
2. All broad and deep mathematical concepts can and should be expressed through **multiple representations**: some notational (arithmetic, algebraic, tabular, etc.), others geometric (on the line, in the plane, through diagrams), and still others, linguistic. Teaching mathematics entails making constant comparisons between students' representations and conventional representations introduced through instruction.
3. Problems about **quantities** (weight, cost, area, length, time, etc.) often raise issues distinct from problems about **numbers** that have a similar mathematical structure. (For example, sharing n objects among d people raises issues that do not arise, in the same form, when one divides the integer n by the integer d.)

The concept of function is central in modern mathematics. Although functions are traditionally formalized only in high school, they can be approached from earlier grades as underlying much of the curriculum content, including not only algebra but also the operations of arithmetic, the study of fractions, rations, proportions, and geometry and as tools for fitting data to models. While recognizing that each field had slightly different views on functions, how they were used, and why they were important, there was a strong consensus that functions would offer a basis for substantial contributions from the three fields.

Section 3: Explanatory framework:

Our conceptual framework evolved from more than a year of discussions and project planning. It reflects our view that the teaching of mathematics requires respect for mathematical concepts and definitions, sensitivity about how students and teachers make sense of it, and consideration of its applications. Maintaining an eclectic perspective has been a constant concern throughout the development of the Institute.

In our program, teachers learn about mathematics content and applications that go beyond what they teach. They consider the perspectives of mathematics, physics, and mathematics education, examine students' mathematical learning and reasoning in videotaped classrooms, and conduct interviews with students to obtain fresh insights and evaluate how issues raised in the courses would play out in their classrooms.

Because teachers must contend with demands from these three perspectives, we have found it useful to acknowledge clashes that invariably arise—for example, cases in which the interpretation of a problem will hinge crucially on whether one treats the objects of concern as pure numbers or as physical quantities, or cases in which the patterns students readily identify in tables of data compete with the assignment rules that are expected to be the aim of a lesson.

Our initial proposal, to offer three courses on (a) functions and their representations, (b) transformations and their use in the solution of equations, and (c) change as modeled by functions, has been constantly reviewed, expanded, and adapted.

The first course offered to the first cohort of teachers dealt with functions and relations, functions on the real number line, and representation of functions on the plane. Each of the 14 weeks of the course primarily focused on either mathematics, education, or science, with eight weeks on the mathematics of functions and relations, two weeks on mathematical modeling in science, and four weeks focused on teaching and learning. Teachers worked in online teams of six teachers, with two instructors (one educator and one mathematician or physicist) as tutors. For each of the mathematics or science weeks, teachers first worked on exploratory and in-depth notes and videos on mathematical content or uses of mathematics in science contexts, and considered short essays presenting a mathematician's, a scientist's, and an educator's perspective on each unit topic. Teachers commented on the work of their online team peers and were encouraged to discuss topics related to mathematics or classroom practice in a general online forum.

As the course evolved, we realized that the integration among the three disciplines, although present, was not fully achieved. We also realized that the teachers were particularly drawn to the "education weeks," for which they analyzed video of classroom activities or samples of student work produced by the early algebra previous research. If the Poincare Institute was to have a real impact, teachers should be applying what they learned from the courses into their classroom. At the same time, mathematical content should not be shortchanged. This called for a better integration of the educational activities into the mathematics and science content of the courses.

The course structure was then revised to include four three-week units, each including mathematics, science, and educational views, interweaved throughout the written and videotaped materials. In the first week of each unit teachers explore the topic, discuss models of teaching the unit's specific subject, analyze students' ideas and challenges in learning the subject, and solved mathematics and pedagogical problems relevant to their learning and teaching. In the second week, they develop a deeper understanding of the mathematical content of the unit, again through notes, videos, problem solving, and online discussions, working on assignments that would require them to think through the questions often from the points of view of mathematics, science, or education. Then, in the third week, groups of two to three teachers jointly design a learning activity for possible future implementation, based on topics from the previous two weeks. In their final individual project for the course, each teacher implemented in their classroom a learning activity they had planned. They videotaped this activity and analyzed his/her teaching and their students' learning in a short individual report, posted online along with selected classroom video clips, and discussed by other teachers.

Adopting the new course structure, the planning of the three courses offered to the second cohort of teachers started with a long period of discussions where all members of the Poincaré Institute team provided suggestions regarding the content to be addressed. Units were rewritten by small interdisciplinary teams and an Editorial Board composed by a mathematician, a physicist, and an educator, acted as coordinators of the process gathering feedback from all members of the Poincaré Institute until course materials were considered as ready for online presentation.

In our presentation we will exemplify the interdisciplinary character of our current work with examples of course materials and activities as they are now offered to the second cohort of 64 teachers.

Section 4: Discussion:

Creating interdisciplinary collaboration demands time and commitment, as well as openness towards views that may differ from one's own. Cross-disciplinary clashes about how to proceed are unavoidable, sometimes very inconvenient, but indispensible for improving the Institute. They may lead to different proposals for course activities and, finding a common, coherent ground, may demand hours of discussion, dozens of e-mails, and multiple drafts. Sometimes no agreement is reached and, when it happens, we have found it fruitful to propose that the different views or activities are explicitly presented and judged by the teachers themselves. With participants that are also involved in many other teaching and research activities, this may prove to be a difficult or even impossible task. But if you can find the time to commit to it, it is a most rewarding endeavor.

Section 5: How will you structure this session? What is your plan for participant interaction?

We will start by asking members of the audience to indicate their own principal field of training and to order in importance, on a scale of 1 to 4, the following priorities for improving middle school mathematics education:

(a) That teachers develop a strong and deep mastery of mathematical content.
(b) That teachers become knowledgeable regarding how students reason about and approach particular mathematics topics.
(c) That teachers become aware of the connections between their curricula and the mathematics standards they are expected to meet.
(d) That teachers learn to devise engaging and challenging problems for students to solve.

We will show how each of these perspectives contributes to the design of the Institute courses and activities. We will also highlight special instances in which what may have seemed like a sensible way to proceed, from within a particular perspective, needed to be re-evaluated given concerns that arose from another perspective.