**Session Title:**
Learning to Measure our Impact: Focus on Mathematics

**MSP Project Name:**
Focus on Mathematics, Phase II: Learning Cultures for High Student Achievement

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**Feedback Session**

**Strand 3**

**Summary:**
Focus on Mathematics is a targeted MSP funded by the National Science Foundation since 2003. As part of this work, we are developing a research program with the goal of understanding the connections between secondary teachers’ mathematical habits of mind (MHoM) and students’ mathematical understanding and achievement. We are developing tools to study the question: What are the MHoM that high school teachers use in their professional lives and how can we measure them? In this feedback session, we will share preliminary versions of our observation protocol and sample items from a paper and pencil assessment of MHoM. We will also share our working definition of MHoM.

**Section 1: Description of product, tool, process, curriculum, or instrument:**
Building on close to two decades of prior work, the Focus on Mathematics (Phase 1 and Phase 2) MSP has, over the last decade, developed and refined a distinctive framework for a mathematics-centered approach to developing teacher leaders and it has built a mathematical community based on that framework. The Focus on Mathematics approach involves teachers, mathematicians, and mathematics educators, all working in programs that put mathematics at the core of a tightly connected set of professional development activities. These programs include intense immersion experiences in mathematics and school-based study groups that involve teachers and higher education faculty. These activities, taken either individually or in tandem, are examples of what we will call a mathematical learning community.

We believe that participation in a mathematical learning community produces career-changing improvements in teaching practice. We have evidence of significant positive impact both to the 5–12 community and to the mathematicians and their teaching at the post-secondary level. As our sense of these mathematical learning communities matured,
refinements to the approach and new questions became apparent; as the communities
themselves matured, new information became available. Accordingly, we began a second
phase of work to deepen the community of the existing partnership, to scale up the work,
and to conduct research on some questions that have emerged that are of considerable
interest to the field. In particular, we have been studying mathematical knowledge for
teaching.

The notion of mathematical knowledge for teaching (MKT) has been studied by many
researchers. Our understandings of this notion parallel much of what we have read in the
literature, but are based on our particular experiences over the past 20 years, as
mathematicians engaged in doing mathematics with secondary teachers. As one part of
the work of Focus on Mathematics, we proposed to develop a research program with the
long-term goal of understanding the connections between secondary teachers’
mathematical knowledge for teaching and secondary students’ mathematical
understanding and achievement.

We recognize the need for a scientific approach to investigate our beliefs about the
importance of participation in a mathematical learning community, and more broadly,
about the importance of teachers’ development of mathematical habits of mind (as
described below). Our current work is focused on developing instruments to carry out
these investigations. These instruments include a **paper and pencil assessment** designed
to measure how teachers think about mathematics for themselves, and an **observation
protocol** designed to measure how teachers use those habits in a classroom setting.

**Distinguishing Features of Our Instruments:**

Other researchers are developing instruments to assess secondary teachers’ content
knowledge and use of mathematics in their classrooms. These efforts will all contribute to
what we know about MKT, but there are two distinguishing features of our work:

- A focus on *Mathematical Habits of Mind* (MHoM)—the methods and ways of thinking
  through which mathematics is created—rather than on specific results [3]. It’s
  impossible, even in three or four years of high school mathematics aligned with the
  Common Core, to equip students with all of the facts they’ll need for college and career
  readiness. But learning to think in characteristically mathematical ways is a ticket to
  success in fields ranging from business, finance, STEM-related disciplines, and even
  building trades.
- The core involvement, at every level, of mathematicians who have thought deeply about
  the implications of their own MHoM for precollege mathematics curricula, teaching, and
  learning.

Our instruments are therefore aimed at discerning the extent to which secondary
classrooms are centered on the practice of *doing mathematics* rather than on the special-
purpose methods that often plague secondary curricula. In our work with teachers over
the past decades, we’ve seen how expert teachers use core mathematical habits of mind in
their profession—in class, in lesson planning, and in curricular sequencing. And, as the
Common Core becomes the nationally accepted definition of school mathematics,
teachers will be expected to make the development of mathematical habits an explicit part of their teaching and learning agenda.

In this session we will ask for feedback on three aspects of our work:

1. **We will ask for feedback on our detailed definition of MHoM.** To operationalize the MHoM concept, we relied on our own experiences as mathematicians doing mathematics with secondary teachers [11]. We also studied existing literature—in particular, Dewey’s earlier treatment of habits and habits of mind [5, 6], the Study of Instructional Improvement (SII) and the Learning Mathematics for Teaching (LMT) projects to develop measures of mathematical knowledge for teaching for elementary teachers [1, 2, 7, 8], and Cuoco, Goldenberg, and Mark’s description of mathematical habits of mind [3, 4]. And we consulted the national standards, i.e., the NCTM Principles and Standards for School Mathematics [9] and the Common Core Standards for Mathematical Practice [10]. We also surveyed mathematicians. But above all, we went into the classrooms of Focus on Mathematics (FoM) teachers, where we saw a broad sampling of MHoM strengths—some exhibited precise use of language and careful reasoning skills; others had strong exploration skills, were good at designing mathematical experiments, or showed special strength at generalizing from concrete examples. From these various sources, we zoomed in on two categories of mathematical habits of mind: (1) Seeking, Using, and Describing Structure and (2) Using Mathematical Language.

2. **We will ask for feedback on our pilot paper and pencil assessment.** We are developing a paper and pencil assessment that measures how secondary teachers use MHoM in their own doing of mathematics. To date, we have gone through several rounds of design, pilot-test, data analysis, and revision of this instrument.

3. **We will ask for feedback on our pilot observation protocol.** In the Focus on Mathematics Phase II (FoM-II) study, we are also designing a pilot observation protocol that measures teachers’ use of MHoM in their classroom instruction. An important feature of our observation protocol is that it measures how teachers use their MHoM in their instruction. Thus teachers are coded not for possessing certain mathematical habits in the abstract, but for choosing to bring them to bear in a classroom setting.

**Using Feedback:**
We will share several items from the assessment and a sample rubric from the observation protocol. Feedback from this session will inform the development of the next pilot version of the assessment and the next version of the protocol.

**References:**


**Section 2: Question, issue, or challenge that is the primary focus of the session:**

How can we measure secondary teachers’ knowledge and use of mathematical habits of mind?

**Section 3: Types of people who you think might be most interested in discussing this and offering feedback:**

We welcome feedback from project directors, PIs, mathematicians, education researchers, evaluators, K-12 administrators and teachers.

**Section 4: How will you structure this session? What is your plan for participant interaction?**

In this session, we will begin with a short (5 minute) introduction to our research project, including information about past field tests. We will also share our rationale for our current choice of MHoM.

We will share sample assessment items and rubrics and give participants an opportunity to try out the items and rubrics (10 minutes). After giving participants an opportunity to give general feedback and ask questions, we will use the following questions to frame a discussion around the assessment items (25 minutes):

- Is the mathematical content of the items relevant to secondary teaching?
- Is the mathematics accessible to secondary teachers?
- Were the items interesting enough to engage you in mathematical thinking?
- Do the items seem to measure the MHoM we are trying to measure?
• Are the rubrics “usable”?
• What other kinds of information would you have liked to see included in the rubrics?

We will then hand out part of our observation protocol and show a short video of a secondary teacher (8 minutes). Working with a transcript, we will ask participants to try out the observation protocol (7 minutes). After giving participants an opportunity (again) to give general feedback and ask questions, we will use the following questions to frame a discussion (30 minutes):
• Were you able to use this protocol to capture the teachers’ use of this particular MHoM?
• What else would you have liked to see in our protocol?
• How could we make our protocol more user-friendly?
• Are our MHoM definitions, our protocol, and our assessment well-aligned with each other?

We will end with a discussion about next steps and opportunities for other MSPs to participate in our research study or use the instruments.