Session Title:
Implementing Pathways Curriculum: What Are We Learning from In-Service and Pre-Service Teachers to Inform and Modify Project Resources?

MSP Project Name:
Pathways to Calculus

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Project Session

Strand 2

Summary:
Our project's findings from Phase I and II interventions provide two keys important for implementation of the Pathways curriculum – (i) teachers must have a deep understanding of the mathematics they teach and (ii) student thinking is vital for teachers to focus on as they work with students in their classroom. In this session we present findings about how in-service and pre-service secondary mathematics teachers use the Pathways curriculum and how these data inform our work across the two populations (in-service and pre-service). This session will engage participants in activities that reflect our findings from working with both populations with the intentions of generating productive discourse about how working with multiple populations can serve project goals.

Section 1: Questions framing the session:
1) What shifts in Pathways teachers' (pre-service and in-service) content knowledge are achieved from using Pathways curriculum (teacher supports and student materials)?
2) How does working with either population (pre-service or in-service) inform (in different ways) our work with the other population?

Section 2: Conceptual framework:

According to Bryan (1999), "A number of recently completed studies of the knowledge of prospective secondary mathematics teachers have shown that prospective secondary mathematics teachers, even with a substantial amount of university mathematics coursework completed, still may not have a level of conceptual understanding of their future subject matter that seems requisite for the teaching of that subject matter, especially in ways consistent with those advocated by the current reform movement in mathematics education" (p.10). Our own research, during Phase I and continuing during Phase II, further illustrates that practicing secondary teachers often do not possess *key developmental understandings* (Silverman & Thompson, 2008; Simon, 2006) of algebra and precalculus that are foundational for the teaching of these courses. In fact, the mean score of over 250 secondary mathematics teachers (from 5 large school districts) on the Precalculus Concept Assessment was 13 (out of 25). A score of 13 out of 25 is what one might expect of a moderately achieving precalculus student at the end of their course (Carlson, Oehrtman, & Engelke, 2010).

A number of studies involving pre-service secondary mathematics teachers have shown similar results–that prospective secondary mathematics teachers may not have a level of conceptual understanding that supports students learning key mathematical ideas that are needed for continued mathematical learning and their futures as teachers (Bryan, 1999; Bush, Lamb, & Alsina, 1990; Even, 1993; Wilson, 1994). Even and Tirsoh (1995) argued that teacher knowledge of content within the secondary curriculum (e.g., functions and undefined mathematical operations) is not sufficiently comprehensive and articulated for teaching. Cooney, Shealy, and Arvold (1998) found that pre-service secondary mathematics teachers lack fundamental understandings of school mathematics despite their success in studying advanced university level mathematics. Bryan (1999) reported that pre-service secondary mathematics teachers exhibited a "conceptual void" for central ideas that they would soon be required to teach (e.g., division by zero, division by a fraction, transformations of the graph of an equation).

Contributing insights into the implications of teachers' lack of key developmental understandings, Cai (2006) found that when U.S. secondary teachers used examples in the classroom, they tended to focus on the procedural aspect of dealing with the example's content. That is, U.S. teachers talked about the specific features of the example and how those features fit into a solution strategy. However, Chinese teachers, unlike their U.S. counterparts, often use examples to make general points by discussing the underlying concepts that the examples were to exemplify. This difference points to a feature of the U.S. mathematical culture that teachers in our project often struggle to overcome—teaching mathematics is about helping students learn to answer questions and examples are used to teach procedures; to teachers, examples are often not thought about in terms of a key developmental understanding.

Extending research on the teaching of mathematics, Wagner, Speer and Rossa (2007) studied two Ph.D. mathematician's attempts to teach differential equations using a student-centered approach. They documented the significant obstacles that the mathematicians met when attempting to take student thinking into account, and concluded that their personal image of what is involved in developing an understanding of a mathematical idea (based in logical and mathematical structure) did not align with how students' thinking developed, and that the mismatch was a significant source of difficulty when the mathematicians considered how they might adjust their instruction to account for students' learning difficulties that the mathematicians discerned but did not understand. In our language, we would say that the mathematicians' were unable to decenter from their mathematics to imagine possible ways that students might have understood the material and their instruction.

In an attempt to address the aforementioned difficulties pre-service and in-service teachers face, the research-based Pathways curriculum was developed and informed by theory on learning the ideas of function (Carlson 1998, 1995), the processes of covariational reasoning (Carlson et al. 2002), and literature about mathematical discourse

(Clark et al. 2008) and problem-solving (Carlson and Bloom 2005). The curriculum contains modules based on research of student learning and conceptual analysis of the cognitive activities conjectured to be necessary to understand and apply the module's central ideas. The curriculum also supports a problem solving approach to mathematics, where students are expected to reason and construct their own understanding. To support teachers' focus on student thinking, common misconceptions students have about the targeted concepts are used as a way to elicit student thinking and reason about the mathematics content.

Our project's findings from both Phases I and II interventions provide two keys important for implementation of the Pathways curriculum – (i) teachers must have a deep understanding of the mathematics they teach and (ii) student thinking is vital for teachers to focus on as they work with students in their classroom. As teachers involved in our project encountered difficulties in these two areas, we have worked to improve research and implementation efforts to better understand and address these difficulties. This has led us to extend project interventions to the undergraduate pre-service level, while coordinating these efforts with in-service interventions.

For Pathways teachers to have a deep understanding of the mathematics they teach, they need to first understand the underlying important mathematics concepts in each Pathways lesson. By coming to understand these underlying concepts, the teachers can then plan cognitive demanding lessons that require students to apply their understanding of a mathematical concept rather than just carry out a procedure. We use the *fundamental mathematics concept* (FMC) to refer to a mathematical concept that is a fundamental building block of the learning goals that drive the selection and implementation of mathematical tasks for a lesson. In our work with our in-service teachers, we have found that their meanings for various mathematics topics typically stand in opposition to their identifying FMCs consistent with those around which the Pathways materials were designed. While we have gained some insights into how in-service teachers' mathematical meanings influence their implementation of Pathways materials, our data reveals that teachers' images of current U.S. schooling present obstacles in shifting their practice to be more inquire based and conceptually focused. Responding to the need of better understanding the conceptual difficulties that teachers face, we modified some of the Pathways materials for use in undergraduate education content courses for pre-service secondary teachers. In this context, we are simultaneously investigating pre-service teachers' meanings of central secondary mathematics topics and how to engender shifts in these meanings that are consistent with Pathways goals. In turn, findings from such investigations are informing improvements to the Pathways curriculum and interventions in order to realize improved shifts at the in-service level.

The second key aspect of teaching with the Pathways curriculum is focusing on student thinking. Our work with in-service teachers revealed that they have little experience considering student thinking and its importance in developing student meaning. The Pathways curriculum is designed to elicit student reasoning; however, if teachers are not focused on student thinking they may miss opportunities to help students engage in productive thinking and build critical connections. As we continue to make strides in understanding how to best support in-service teachers transitioning their practice to be more attentive to student thinking, we are using this knowledge to incorporate a focus on student thinking in the preparation of pre-service teachers. And, by working with pre-service teachers on understanding student thinking, we are gaining insights into how various aspects of an individual's mathematical knowledge influence their ability to productively interpret student thinking.

Section 3: Explanatory framework:

In-service teachers: Remillard (1996, 2000) found minimal teacher learning resulted from teachers using curriculum guides. Rather "the most significant learning occurred during teachers' processes of enacting curriculum in the classroom. Teachers' ideas about mathematics, teaching, and learning were challenged and altered when they examined unfamiliar mathematical tasks and interpreted students' work while teaching" (Remillard & Bryan 2004, p. 355). When teachers use the Pathways materials they encounter a similar perturbation. Our preliminary data suggests that the transition for in-service secondary mathematics teachers is initially challenging for all teachers. Over the past year we have scaled Pathways to all teachers within two large school districts in the southwest, and all teachers in high schools across three smaller school districts. All teachers attend a three-day professional development workshop prior to using the materials that first engages these teachers as students. This is necessary because the secondary teachers have not previously been supported in understanding key mathematical ideas in their curriculum. The workshop leaders implement the curriculum with the teachers by working through student tasks in groups of 3-5 teachers. Individual groups then explain their solution approaches and the thinking that lead to their final product. At the completion of the workshop teachers are given multiple resources (e.g., detailed instructor notes for cognitively scaffold in-class tasks, powerpoints, and detailed answer keys for all tasks and homework) to use in the classroom.

Our work with in-service teachers has revealed that most teachers prepare their lessons as they have always done, reading over the instructor notes and rarely complete the task and homework that students will complete for the day. Yet, as they implement the Pathways tasks in their classrooms they find it difficult to answer student questions because they have not spent time to understand the FMC(s) of the lesson for themselves prior to teaching the lesson. They also have not thought through different ways of student thinking that may be valid, yet different from their own. These findings have led us to investigate how we might support our in-service teachers in (i) valuing understanding or teaching mathematical content in a way that differs from how they were taught and (ii) considering the processes involved in understanding an idea and the different ways of thinking that students may express.

Pre-service teachers: Quantitative reasoning (Thompson, 1990) forms a central content strand in the Pathways curriculum. Throughout the Pathways curriculum, teachers are expected to understand the role of quantitative reasoning in the learning of various mathematical topics, and it is this area that pre-service and in-service teachers have the most difficulty. To gain insights into why teachers face difficulties in identifying the role of quantitative reasoning in the Pathways curriculum, our project incorporated a series of studies with pre-service teachers, including several teaching experiments.

A main finding from these studies is how a focus on mathematical conventions often produces meanings that are incompatible with quantitative reasoning. As one example, when presented with a graph of $y = 3x$ such that y is graphed on the horizontal axis and x is graphed on the vertical axis, a majority of the pre-service teachers claimed that the correct relationship is not graphed. A majority of these pre-service teachers also claimed that a student who graphed $y = (1/3)x$ with y on the vertical axis and x on the horizontal axis "more correctly" graphed $y = 3x$. The pre-service teachers' responses have several implications in the context of their mathematical knowledge and focus on student thinking. First, the pre-service teachers' responses on tasks like this one indicate that their understandings inherently involve certain mathematical conventions. Thus, when work is presented to them that is quantitatively correct (e.g., graphing $y = 3x$ as first described), their understandings lead them to claim that such work is incorrect, as opposed to reasoning quantitatively to determine that the graph conveys the correct relationship. Second, the pre-service teachers' responses indicate that the place of mathematical conventions in their understandings inhibits how they might interpret student work. During each interview task, the pre-service teachers' first judged the posed work against mathematical conventions, as opposed to attempting to discern a viable way of thinking behind the posed work. For instance, one pre-service teacher claimed, "he's missing the whole concept of a graph." Collectively, these findings indicate the connected role of quantitative reasoning and interpreting student thinking, and we have learned that a more increased focus on central principles of quantitative reasoning is needed when working with both in-service and pre-service teachers, especially in the context of interpreting student thinking.

To attend to the need for pre-service teachers to have a deep mathematical understanding of the content they will teacher, we engage them as both students and teachers in working through two Pathways modules (Reasoning About and Representing Quantitative Relationships and Unit Circle Trigonometry). Pre-service teachers design lesson plans and then teach these to their peers. A major goal of the course is for pre-service teachers to identify the Fundamental Mathematics Concept (FMC) of a lesson prior to designing their lesson plan. We have found that pre-service teachers typically rely on how they learned mathematics when they begin to identify FMCs – therefore, their descriptions are initially more procedural and they are unable to either identify or explain the FMC of the lesson. Pre-service teachers tend to describe what students will be able to do, (e.g., use a formula, compute a number). For example, one group of students wrote the following FMC at the beginning of the semester for a lesson entitled *Proportional Relationships* "A relationship is proportional if when y/x=m regardless of what x and y are, y/x will always be m. Even though there is a constant relationship in the change between two quantities, that doesn't necessarily mean that the two quantities are proportional to each other. When we know the proportion and one quantity, we can tell what the second quantity is in relation to the first which leads to a linear representation in a graph". These students' meanings for proportional relationships consisted of a procedure. However, after project members met with the group of pre-service teachers and pushed them for an understanding of proportionality that did not rely on a procedure, they determined the following FMC – **"**A relationship is proportional if two quantities have a common ratio, can be scaled, or have a common multiple. You can verify two quantities are proportionate as long as the common ratio, scale factor, or common multiple holds for all the data. If the quantities are proportional, any of the methods will verify this". As displayed in these two contrasting FMCs, we are attempting to focus our pre-service teachers on identifying FMCs that consist of quantitative reasoning and foundational meanings for students prior to teaching a lesson.

By investigating pre-service teachers' ability and quality of verbalizing FMCs, we have found that improvement is supported by three critical factors – (1) access to conceptually-oriented curriculum that requires them to focus on unfamiliar mathematical tasks and question their own mathematical understandings; (2) the opportunity to identify and write FMCs for multiple lessons over the course of the semester and then reflect on these experiences; and (3) the ability to see how identifying the FMC of a lesson provides a roadmap for preparing a lesson that is focused on meaning making.

Section 4: Discussion:

As our project continues to move forward, our findings from both in-service and pre-service populations are informing project materials and resources. Mostly notably, our work with pre-service teachers is identifying where efforts are best placed in terms of influencing teachers' content knowledge in ways consistent with the Pathways goals. Such knowledge will inform future intervention efforts (e.g., professional development and teacher curriculum notes) in an attempt to improve implementation of Pathways materials. Likewise, as we determine how to better support in-service teachers in focusing on student thinking, we are modifying pre-service materials to reflect these findings. For instance, it is key that teachers determine how students' ways of thinking are viable to the student, regardless of the mathematical correctness of the students' thinking. Thus, at the pre-service level, we are designing activities that prompt discussion of students' ways of thinking in ways that diminish a focus on judging these ways of thinking. Instead, the pre-service teachers are expected to discuss the mathematical structure of these ways of thinking and the implications of such of ways of thinking for the students' learning.

The discussed work will be relevant to other MSPs in several ways. First, in the context of research-based curriculum, we illustrate novel findings relative to supporting teachers' implementation of Pathways materials in ways that elicit and support student reasoning. Particularly, we identify the interrelated roles of content knowledge and focusing on student thinking in supporting such implementation. Second, our evolving model of connecting in-service and pre-service interventions will offer ideas for other MSP projects to consider. We will describe how implementing project materials with both populations can help further research and implementation efforts of the project. Most importantly, we posit that using compatible materials with both populations creates opportunities to incorporate several interrelated research foci at an in-depth level.

Section 5: How will you structure this session? What is your plan for participant interaction?

We will begin the presentation with a short overview and background on Project Pathways MSP (Phase I and II) to provide a context for the data being shared. We will then present data results from our pre-service and in-service teachers who "use" the Pathways curriculum. During this portion of the presentation, participants will engage in some of the tasks that we give our pre-service and in-service teachers so they can better understand the results we are sharing. We will conclude the presentation with a discussion on how other MSP projects view the potential of working with both in-service and pre-service populations and sharing lessons we have learned from the two populations and how each population is informing our work with the other.